Simultaneous Determination of Orbital Parameters and Maneuver Impulse

V.A. Stepanyants, A.G. Tuchin[1], S.Yu. Kamensky[2], Z.N. Khutorovsky[2]

[1] M.V. Keldysh Institute of Applied Mathematics of Russian Academy of Sciences
[2] JSC “MAK Vympel”

An important part of the problem of tracking objects on low Earth orbits is a problem of automatic detection, qualitative and quantitative estimation of the executed maneuvers. This report contains an algorithm for simultaneous determination of orbital parameters and impulse parameters from measurements. A model of an instantaneous impulse execution is considered. Nine parameters are determined simultaneously: space vehicle (SV) state vector before the impulse and the impulse parameters. In this algorithm the impulse application moment is considered to be predetermined. The moment is determined by the algorithm specified in the preprint [1]. The examples of the algorithm application are considered. The proposed algorithm is used in processing of measurements of Scientific Network of Optical Instruments for Astrometric and Photometric Observations (NSOI AFN3).

Changes of orbital parameters caused by reactive forces, in case there are measurements before and after the impulse, can be interpreted as a single-impulse maneuver. This is the most frequently met case. If orbital parameters were determined more frequently than the maneuvers are executed, all orbit measurements could be interpreted as single-impulse maneuvers.

If a single-impulse maneuver takes place, the trajectories before and after the impulse application shall intersect in a point where the impulse application occurred. That is why when analyzing possible reasons for a change of orbital parameters, it is necessary to find a point at which the distance between the trajectories before and after the impulse application is minimal [1]. The moment of time determined under the algorithm described in [1] is an input parameter for the algorithm for determination of the impulse vector and the SV orbital parameters.

Let’s consider the algorithm which determines the impulse parameters along with the state vector. This algorithm, as well as in the common case [2], determines the parameters on the basis of minimization of the following functional type:

$$J=\sum_{i=1}^{N}\left(\left(\mathbf{\Psi}_i\right)_{\text{наб}}-\mathbf{\Psi}_i\left(t_i,\mathbf{x}\left(t,\mathbf{q}\right)\right)\right)^T\mathbf{R}_i^{-1}\left(\left(\mathbf{\Psi}_i\right)_{\text{наб}}-\mathbf{\Psi}_i\left(t_i,\mathbf{x}\left(t,\mathbf{q}\right)\right)\right),\qquad(1)$$

where

$\left(\mathbf{\Psi}_i\right)_{\text{наб}}$ – the measured values vector in the moment of time t_i;

$\mathbf{\Psi}_i\left(t_i,\mathbf{x}\left(t,\mathbf{q}\right)\right)$ – the calculated value of the measured parameters vector;

$\mathbf{x}\left(t,\mathbf{q}\right)$ – the functional dependence of the state vector in the moment of time t on the determined parameters vector $\mathbf{q}$;

$\mathbf{q}$ – the determined parameters vector including the space object state vector in the moment of the last measurement and the impulse vector in the moment of its execution (a total of 9 parameters);

[3] International Scientific Optical Network (ISON)

$\mathbf{R}_i$ – the priori covariance matrix of errors of the *i*-th measurement.

If the a priori value of the impulse vector $\boldsymbol{\Delta}\mathbf{v}_{\mathrm{A}}$ and the covariance matrix of that a priori value $\mathbf{P}_{\Delta V}$ are also known, the following member can be added to the functional (1):

$$\left(\boldsymbol{\Delta}\mathbf{v}_{\mathrm{A}}-\boldsymbol{\Delta}\mathbf{v}\left(t_{\text{имп}},\mathrm{q}\right)\right)^{\mathrm{T}}\mathbf{P}_{\Delta V}^{-1}\left(\boldsymbol{\Delta}\mathbf{v}_{\mathrm{A}}-\boldsymbol{\Delta}\mathbf{v}\left(t_{\text{имп}},\mathrm{q}\right)\right), \tag{2}$$

which makes it possible to take into account the a priori information when determining the parameters vector $\mathbf{q}$.

The functional minimization is carried out iteratively by the Newton method. In this case the correction vector $\boldsymbol{\Delta}\mathbf{q}$ to the components of the determined vector $\mathbf{q}$ at each stage of the iteration process minimizes the following function:

$$J_L=\sum_{i=1}^{N}\left(\mathbf{H}_i\boldsymbol{\Phi}_{\mathrm{ext}}\left(t_i\right)\boldsymbol{\Delta}\mathbf{q}-\mathbf{z}_i\right)^{\mathrm{T}}\mathbf{R}_i^{-1}\left(\mathbf{H}_i\boldsymbol{\Phi}_{\mathrm{ext}}\left(t_i\right)\boldsymbol{\Delta}\mathbf{q}-\mathbf{z}_i\right), \tag{3}$$

where

$\mathbf{H}_i$ – the matrix of partial derivatives of the measured function $\boldsymbol{\Psi}_i\left(t_i,\mathbf{x}\left(t_{i,sc},\mathbf{q}\right)\right)$ under the SV state vector components $\mathbf{x}$ in the moment of time $t_{i,sc}$ corresponding to the measurement registration time t_i with consideration of the signal travel time from the SV to the observer;

$\boldsymbol{\Phi}_{\mathrm{ext}}\left(t_{i,sc}\right)$ – the matrix of partial derivatives of the state vector components $\mathbf{x}$ under the components of the determined parameters vector $\mathbf{q}$ in the moment of time $t_{i,sc}$;

$\mathbf{z}_i=\left(\boldsymbol{\Psi}_i\right)_{\text{наб}}-\boldsymbol{\Psi}_i\left(t_i,\mathbf{x}\left(t,\mathbf{q}\right)\right)$ – discrepancy between the calculated and measured values.

The correction $\boldsymbol{\Delta}\mathbf{q}$ is found from the solution of a normal equations system by the formula:

$$\boldsymbol{\Delta}\mathbf{q}=\left(\mathbf{B}^{\mathrm{T}}\mathbf{W}\mathbf{B}\right)^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{W}\mathbf{d}, \tag{4}$$

where

$$\mathbf{B}=\begin{bmatrix}\mathbf{H}_1\boldsymbol{\Phi}_{\mathrm{ext}}\left(t_1\right)\\ \mathbf{H}_2\boldsymbol{\Phi}_{\mathrm{ext}}\left(t_2\right)\\ \dots\dots\dots\dots\\ \mathbf{H}_N\boldsymbol{\Phi}_{\mathrm{ext}}\left(t_N\right)\end{bmatrix},\quad \mathbf{d}=\begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\\ \dots\\ \mathbf{z}_N\end{bmatrix},\quad \mathbf{W}=\begin{bmatrix}\mathbf{R}_1^{-1} & 0 & \dots & 0\\ 0 & \mathbf{R}_2^{-1} & \dots & \dots\\ 0 & 0 & 0 & 0\\ 0 & 0 & \dots & \mathbf{R}_N^{-1}\end{bmatrix}. \tag{5}$$

In order to use the formulas (4) and (5) it is necessary to find the matrix $\boldsymbol{\Phi}_{\mathrm{ext}}\left(t_i\right)$. Let's denote the state vector estimation in the moment of the last measurement t_N by $\hat{\mathbf{x}}_N$. Let's also denote the impulse execution moment by t_{I} and its estimation – by $\boldsymbol{\Delta}\hat{\mathbf{v}}$. Then

$$\mathbf{q}=\begin{pmatrix}\hat{\mathbf{x}}_N\\ \boldsymbol{\Delta}\hat{\mathbf{v}}\end{pmatrix}. \tag{6}$$

Let the SV motion be described by the differential equation:

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(t, \mathbf{x}). \tag{7}$$

Let's denote the differential equation solution (7) in the section from the impulse execution moment t_{I} to the last measurement moment t_N by $\mathbf{x}_{\mathrm{R}}(t)$, and in the interval to the left from the impulse, that is from the first measurement moment t_0 to the impulse execution moment t_{I}, by $\mathbf{x}_{\mathrm{L}}(t)$.

In the impulse application moment t_{I} the vectors $\mathbf{x}_{\mathrm{L}}(t_{\mathrm{I}})$ and $\mathbf{x}_{\mathrm{R}}(t_{\mathrm{I}})$ are connected with the relation:

$$\mathbf{x}_{\mathrm{L}}(t_{\mathrm{I}}) = \mathbf{x}_{\mathrm{R}}(t_{\mathrm{I}}) - \begin{pmatrix} \mathbf{0} \\ \mathbf{\Delta\hat{v}} \end{pmatrix}. \tag{8}$$

Therefore, in the section to the right from the impulse the SV motion and the matrix of partial derivatives comply with the differential equations systems:

$$\frac{d\mathbf{x}_{\mathrm{R}}}{dt} = \mathbf{F}(t, \mathbf{x}_{\mathrm{R}}), \qquad \frac{d\mathbf{\Phi}_{\mathrm{R}}(t)}{dt} = \left.\frac{\partial \mathbf{F}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_{\mathrm{R}}(t)} \mathbf{\Phi}_{\mathrm{R}}(t) \tag{9}$$

on the initial conditions:

$$\mathbf{\Phi}_R(t_N) = \mathbf{E}, \quad \hat{\mathbf{x}}_N = \mathbf{x}_{\mathrm{R}}(t_N). \tag{10}$$

In the section to the left from the impulse the SV motion and the matrix of partial derivatives comply with the equations:

$$\frac{d\mathbf{x}_{\mathrm{L}}}{dt} = \mathbf{F}(t, \mathbf{x}_{\mathrm{L}}), \qquad \frac{d\mathbf{\Phi}_{\mathrm{L}}(t)}{dt} = \left.\frac{\partial \mathbf{F}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_{\mathrm{L}}(t)} \mathbf{\Phi}_{\mathrm{L}}(t) \tag{11}$$

on the initial conditions:

$$\mathbf{\Phi}_{\mathrm{L}}(t_{\mathrm{I}}) = \mathbf{E}, \quad \mathbf{x}_{\mathrm{R}}(t_{\mathrm{I}}) - \begin{pmatrix} \mathbf{0} \\ \mathbf{\Delta\hat{v}} \end{pmatrix} = \mathbf{x}_{\mathrm{L}}(t_{\mathrm{I}}). \tag{12}$$

The matrix of partial derivatives of the current state vector components under the state vector components in the moment of the last measurement $\mathbf{\Phi}(t)$ is described by the following formula:

$$\mathbf{\Phi}(t) = \begin{cases} \mathbf{\Phi}_R(t), & \text{if } > t_{\mathrm{I}}; \\ \mathbf{\Phi}_L(t)\,\mathbf{\Phi}_R(t_{\mathrm{I}}), & \text{if } \;,,\; t_{\mathrm{I}}. \end{cases} \tag{13}$$

It is obvious that the matrix of partial derivatives of the SV state vector to the moment of the impulse application under the impulse vector components is equal to zero. To find the matrix of partial derivatives of the SV state vector to the right from the impulse application moment under the impulse vector components, let's consider the matrix of partial derivatives:

$$\frac{\partial \mathbf{x}_{\mathrm{R}}(t)}{\partial \mathbf{x}_{\mathrm{R}}(t_{\mathrm{I}})} = \frac{\partial \mathbf{x}_{\mathrm{R}}(t)}{\partial \mathbf{x}_{\mathrm{R}}(t_N)} \frac{\partial \mathbf{x}_{\mathrm{R}}(t_N)}{\partial \mathbf{x}_{\mathrm{R}}(t_{\mathrm{I}})} = \left(\frac{\partial \mathbf{x}_{\mathrm{R}}(t_N)}{\partial \mathbf{x}_{\mathrm{R}}(t)} \right)^{-1} \frac{\partial \mathbf{x}_{\mathrm{R}}(t_N)}{\partial \mathbf{x}_{\mathrm{R}}(t_{\mathrm{I}})} = \mathbf{\Phi}_{\mathrm{R}}^{-1}(t)\,\mathbf{\Phi}_{\mathrm{R}}(t_{\mathrm{I}}). \tag{14}$$

Let's denote by $\mathbf{\Phi}_{\mathrm{V}}\left(t\right)$ 6×3 the matrix unit $\dfrac{\partial \mathbf{x}_{\mathrm{R}}\left(t\right)}{\partial \mathbf{x}_{\mathrm{R}}\left(t_{\mathrm{I}}\right)}$ containing three rightmost columns of the matrix. In its construction the matrix $\mathbf{\Phi}_{\mathrm{V}}\left(t\right)$ is a matrix of partial derivatives of the state vector components in the moment of time $t > t_{\mathrm{I}}$ under the impulse vector components.

Combining (13) and (14) we obtain the representation of the matrix $\mathbf{\Phi}_{\mathrm{ext}}\left(t\right)$ in the following form:

$$\mathbf{\Phi}_{\mathrm{ext}}\left(t\right)=\begin{cases}\begin{pmatrix}\mathbf{\Phi}\left(t\right) & \mathbf{0}_{3}\\ \mathbf{0}_{3,6} & \mathbf{E}_{3}\end{pmatrix}, & \text{if} \quad t < t_{I}\\ \begin{pmatrix}\mathbf{\Phi}\left(t\right) & \mathbf{\Phi}_{V}\left(t\right)\\ \mathbf{0}_{3,6} & \mathbf{E}_{3}\end{pmatrix}, & \text{if} \quad t \ldots t_{I}\end{cases}, \qquad (15)$$

where

$\mathbf{0}_{3,6}$, $\mathbf{0}_{3}$	–	zero matrixes 3×6 and 3×3;
$\mathbf{E}_{3}$	–	third order unity square matrix.

In the result of the carried out considerations, an algorithm is made making it possible to obtain the estimation of the impulse and the SV state vector in the moment of the last measurement, using the measurements before and after the impulse and the impulse application moment.

Let's consider the algorithm application by the example of an estimation of an impulse on the space object (SO) in a geostationary orbit (GEO) with the international number 2000-031A. That SO was moved from the point with the longitude of 349 degrees to the point with the longitude of 38 degrees. In the result of execution of the first maneuver on June 18, 2009, the orbital height of the SO was decreased and the SO started drifting to the East with the speed of about 1 degree/day. The braking maneuver, executed on August 2, 2009, provided the SO coming to the point with the longitude of 38 degrees. The SO was located in that point (longitude of 38 degrees) before the beginning of execution of the deviation impulse (August 20, 2009). Let's consider the estimation of the impulse which made it possible to stop the drift to the East under the measurements by Scientific Network of Optical Instruments for Astrometric and Photometric Observations (NSOI AFN)[1]. To obtain the estimation of the breaking impulse it is possible to use the measurements from the moment of finishing of the height decrease impulse execution (beginning of the drift to the East 2009/06/20) to the moment of the beginning of the deviation impulse output (2009/08/20).

At first, under the measurements in the interval from 2009/06/20 to 2009/08/02 we obtain the orbit estimation before the breaking impulse. Then, under the measurements in the interval from 2009/08/02 to 2009/08/20 we obtain the orbit estimation after execution of the breaking impulse. Having two orbits before and after the breaking impulse, it is possible to determine a moment of time when the orbits approach to the minimum distance. In the result of the calculations, the following parameters values were obtained:

Date and time of reaching the minimum distance between the orbits:	2009/08/03 12:16:09.030 UTC;
Minimum distance between the orbits:	86.0 km;
Modulus of the speed vectors difference:	2.89 m/s.

[1] \The Network NSOI AFN is coordinated by M.V. Keldysh Institute of Applied Mathematics

The results obtained can not be called satisfactory due to a big difference between the orbits. In further calculations it is possible to use only the estimation of the impulse execution moment. The bad quality of the estimation is explained by a small value of the measurement interval length, and also, possible additional small impulses which could be executed in that interval, for example, in order to provide orientation or execution of the holding maneuver in the neighborhood of the determined longitude.

To provide a more reliable estimation it is necessary to use the algorithm specified in this report. It shall be noted that the estimation of the impulse application moment obtained under the condition of a minimum distance between the orbits can be considered as the initial approximation when obtaining the estimation of that moment of time. Further this time shall be adjusted using the following criteria:

- minimum of the mean-square deviation of the weighted residual discrepancies:
- minimum of the characteristics speed of the orbital transition impulse;
- minimum of the radial component of the orbital transition impulse.

Let’s explain the last criterion. In real maneuvers the impulse quite rarely has a radial component. It can happen when changing the orbital plane or in emergency situations. In this case the impulse was outputted for acceleration of the SV in order to stop the drift to the East, i.e. in the transversal direction.

The impulse estimation by the criterion of minimum value of the adjusted mean-square deviation (MSD):

Impulse date and time, UTC	2009/08/04 3:01:38.5
Adjusted MSD	2.62
Impulse module, m/s	6.1
Radial component, m/s	5.02
Transversal component, m/s	3.4
Component orthogonal to the orbital plane, m/s	0.4

The impulse estimation, by the criterion of the characteristic speed minimum consumption on condition that the adjusted MSD is not more than the value of 3, makes it possible to obtain the impulse estimation, which better complies with the concept of the SV maneuvering:

Impulse date and time, UTC	2009/08/04 0:01:38.5
Adjusted MSD	2.87
Impulse module, m/s	3.52
Radial component, m/s	-0.40
Transversal component, m/s	3.47
Component orthogonal to the orbital plane, m/s	-0.42

The last estimation agrees with the impulse estimation by the criterion of the minimum modulus of the transversal component on condition that the adjusted MSD is not more than the value of 3.

Conclusions

1. An algorithm is developed making it possible to simultaneously estimate the orbital parameters and the maneuver impulse by the measurements in the interval from and after the impulse execution moment.

2. When estimating the impulse execution time, the criterion of minimum estimation of the characteristic speed as limited by the adjusted MSD used.
3. The reliability of the obtained estimation can be controlled by the value of the radial component of the impulse estimation.
4. A drawback of the proposed algorithm is the big computational resource necessary to obtain the estimation.

The report is made with a partial support by Russian Foundation for Basic Research (RFFI) (project 09-01-00566).

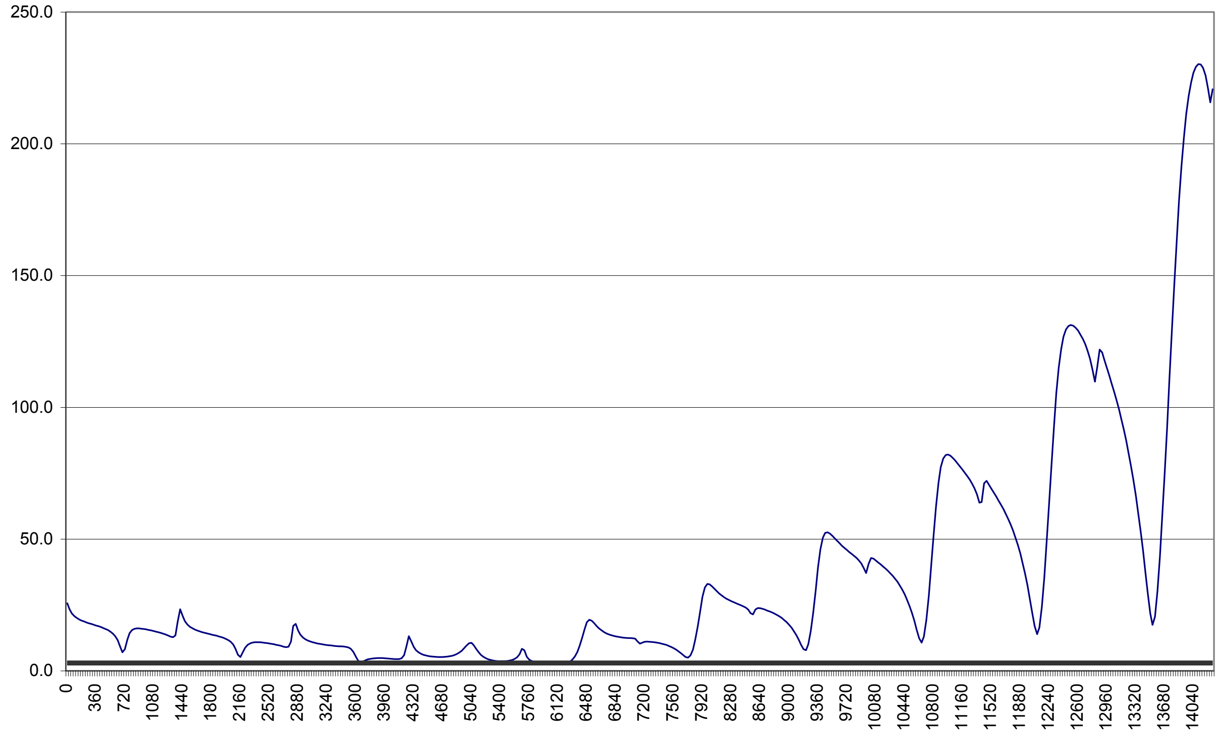

Figure 1. The adjusted MSD depending on the selection of the impulse application moment. On the axis of abscissas the time in minutes is specified from the beginning of the search interval of the impulse application moment. On the axis of ordinates the value of the adjusted MSD is specified (dimensionless value).

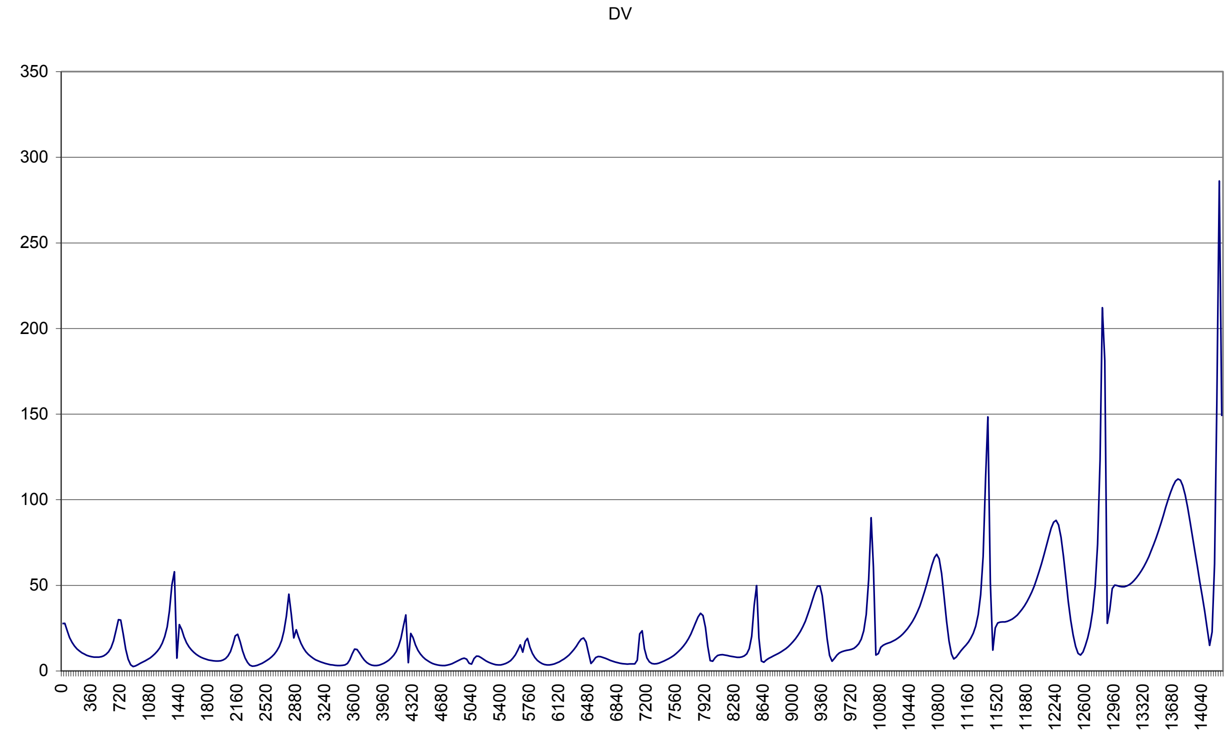

Figure 2. Estimation of the characteristic speed depending on the selection of the impulse application moment. On the axis of abscissas the time in minutes is specified from the beginning of the search interval of the impulse application moment. On the axis of ordinates the value of the characteristic speed in m/s is specified.

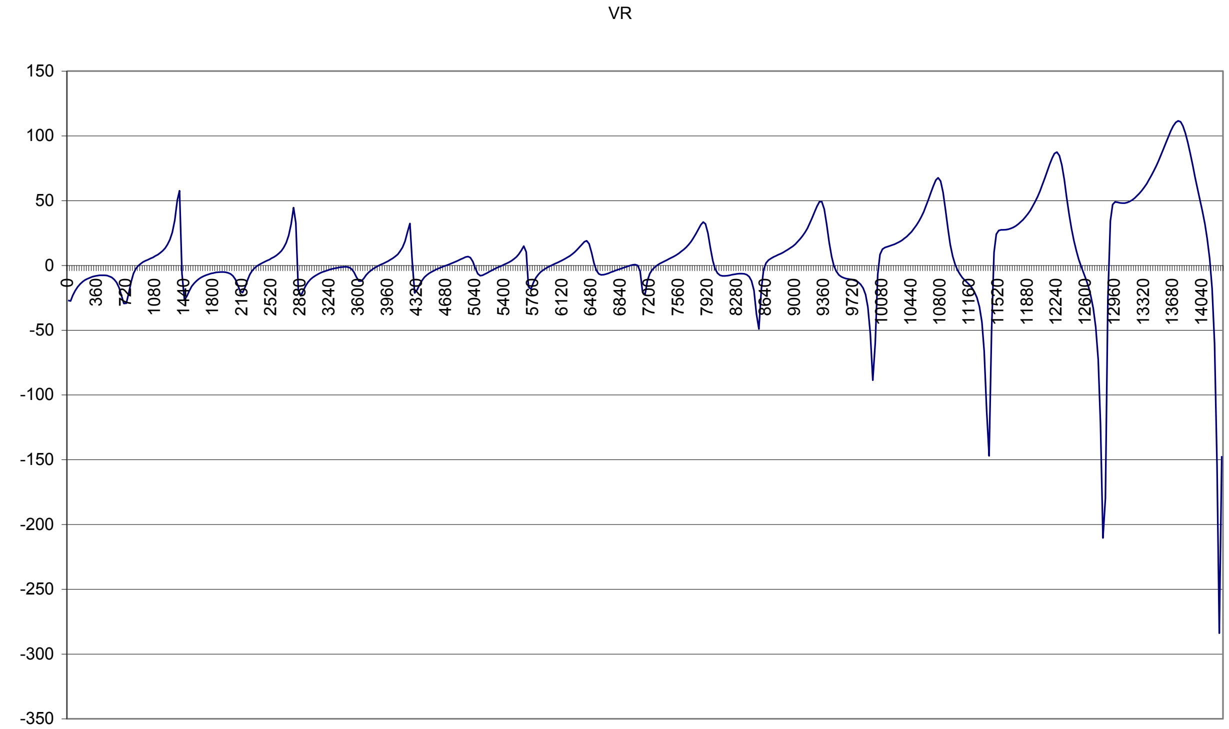

Figure 3. The impulse radial component depending on the selection of the impulse application moment. On the axis of abscissas the time in minutes is specified from the beginning of the search interval of the impulse application moment. On the axis of ordinates the value of the impulse radial component in m/s is specified.

References

1. S. Kamensky, A. Tuchin, V.Stepanyants, Kyle T. Alfriend. Algorithm of Automatic Detection and Analysis of Non-evolutionary Changes in Orbital Motion of Geocentric Objects, AAS 09-103, preprint
2. Akim E.L.., Eneev T.M. Determination of Parameters of a Space Vehicle Motion by the Trajectory Measurements Data, Space Research, 1963. Issue 1, Volume 1