On the Probability of Collisions of Interplanetary Space Vehicles with Meteors

R. I. Kiladze
E. Kharadze Georgian National Astrophysical Observatory, A. Kazbegi Avenue 2a, 0160 Tbilisi, Georgia.
roki@genao.org

Summary. Uncontrolled geostationary satellites suffer, on the average once in 4 years, small sudden changes of speed that are connected with their collisions with fine space debris. Most of these events are caused by collisions with meteoric bodies.

Such collisions also threaten space vehicles, sent to the planets of the solar system with the purpose of research of their physical nature.

The present work is devoted to a definition the risk of collision of a space vehicle with meteors for various possible variants of its interplanetary orbit. Work allows to choose from several possible{probable} variants of a trajectory of interplanetary flight of a space vehicle which is optimum, from the point of view of meteoric danger.

Keywords: Interplanetary flight, meteoric danger.

1. Introduction

Our long-term work with uncontrolled geostationary satellites shows that from time to time (on the average, once for 4 years) they suffer small, within the limits of several mm/s, sudden changes of speed that are connected with their collisions with fine space debris. Most of these events are caused by collisions with meteoric bodies [1 - 5].

Obviously, such collisions, able to lead to accident, also threaten space vehicles (SV), sent to planets of solar system with the purpose of research of their physical nature.

The present work is devoted to a definition of the risk of collision with meteors for various possible variants of SV interplanetary orbit.

As is known, the great bulk of meteoric bodies moves in the form of streams, created by comets. These bodies move along an orbit of the comet generating them, with speeds of some tens of km/s, and do not strongly (within the limits of one million kilometers) deviated from it. Sporadic meteors are of rather small number and move chaotically.

The task of the present work consists of an optimum choice from the set of possible heliocentric orbits of SV, allowing to avoid crossing the orbits of comets (which are a place of a congestion of meteoric bodies) and providing maximal remoteness from them.

It is necessary to note, that distribution of meteoric bodies in streams, more or less, is known only for those few objects, which are crossed by the Earth during its annual movement around of the Sun.

For this reason, it is necessary to suppose a uniformity of distribution of meteoric bodies along orbits. For different streams the identity of their spatial density and the decrease with distance from an orbit of a comet is also postulated.

2. Calculation of risk-factor of collision of a space vehicle with meteors

Further, we shall consider that the spatial density of meteoric bodies with distance from the orbit of a comet, which has generated a meteoric stream, occurs under the formula of Gauss:

(1)

where ρ_Δ is the spatial density of meteoric bodies on distance Δ from an comet's orbit, and ρ_0 –is the spatial density of meteoric bodies in an orbit (a constant for all streams). The parameter Δ_0 is equal 0.01 AU, i. e. 1.5×10^6 km.

For calculation of the value of Δ let us enter the rectangular system of coordinates connected with an orbit of a comet. The origin of the coordinate system coincides with the Sun, X-axis shall direct to perihelion of the orbit and Y-axis in the plane of this orbit, aside movement of the comet.

We shall designate the position of the SV at some moment t in the elected system of coordinates as x_0, y_0, z_0.

The problem of a finding the value of Δ is reduced to finding the minimal distance between SV and the orbit of a comet, i. e. to search for a minimum of function:

$$\Delta^2 = (x - x_0)^2 + (y - y_0)^2 + z_0^2, \qquad (2)$$

where x and y coordinates of points in an orbit of a comet are designated.

From theoretical astronomy it is known:

$$x = \frac{p\cos v}{1 + e\cos v}, \qquad y = \frac{p\sin v}{1 + e\cos v}, \qquad (3)$$

where p is a parameter of an orbit, e - its eccentricity, v - true anomaly.

Equating to zero the result of differentiation of (2) with respect to v, and substituting values x and y, as determined from (3), we receive:

$$(ex_0\cos v_m + x_0 + ep)\sin v_m = y_0[e\cos^2 v_m + (1 + e^2)\cos v_m + e), \qquad (4)$$

where v_m is a value of the true anomaly for a point on the comet's orbit, which is the minimal distance from the SV, .

Inputing a new variable

$$q = tg\frac{v_m}{2} \qquad (5)$$

the equation (4) leads to more simple kind:

$$y_0(1 - e)^2 q^4 + 2(ep + x_0 - ex_0)q^3 + 2(ep + x_0 + ex_0)q - y_0(1 + e)^2 = 0. \qquad (6)$$

The fourth degree of the equation (6) relative to q reflects the fact that, generally, from one point on an ellipse it is possible to drop four perpendiculars, satisfying a condition of an extremum (4).

3. The solution of the equation for the true anomaly

The equation (6) is the most convenient for solving consecutive approximations.

The first approximation to a root of this equation, corresponding to minimal Δ, is found geometrically - crossing an ellipse of a comet's orbit with a bisector of an angle formed by straight lines, connecting SV with focuses of an ellipse.

These lines (the sides of an angle) make some angles, whose values we shall designate as α and β. They are determined by the equations as follows:

$$tg\alpha = \frac{y_0}{x_0}, \qquad tg\beta = \frac{(1 - e^2)y_0}{(1 - e^2)x_0 + 2ep}, \qquad (7)$$

whence the factor k, an inclination of a bisector, is equal:

$$k = tg\frac{\alpha + \beta}{2}. \qquad (8)$$

Hence, the equation of the true anomaly for the intersection of the bisector with comet's orbit is as follows:

$$\frac{p\sin v_0}{1+e\cos v_0}-y_0=k\left(\frac{p\cos v_0}{1+e\cos v_0}-x_0\right), \qquad (9)$$

where $v_{0,}$ the preliminary value of $v_{m,}$ is designated.

Substitution in (9) for the value of v_0, as determined from (5), for the initial (zero) approximation of $q,$ we receive the equation:

$$[(1-e)(y_0-kx_0)-kp]q_0^2-2pq_0+(1+e)(y_0-kx_0)+kp=0, \qquad (10)$$

easily solved relative to q_0.

Because comet orbit's eccentricities are close to unity, the member of the fourth degree in the equation (6) is small, that allows, instead of (6), to consider the equation of the third degree:

$$q^3+3Aq-2B=0, \qquad (11)$$

where

$$A=\frac{ep+(1+e)x_0}{3[ep+(1-e)x_0]}, \qquad (12)$$

$$B=\frac{y_0[(1+e)^2-(1-e)^2q^4]}{4[ep+(1-e)x_0]}.$$

The value of B is specified by consecutive approximations.

If the inequality is carried out:

$$B^2+A^3>0, \qquad (13)$$

equation (11) has one real root, according to formula Kardano, equal to

$$q=\left(B+\sqrt{B^2+A^3}\right)^{1/3}+\left(B-\sqrt{B^2+A^3}\right)^{1/3}. \qquad (14)$$

If the inequality (13) is not carried out, equation (11) has three real roots. In that case the process of approximations consists in the application of iterations:

$$q_{k+1}=(2B-3Aq_k)^{1/3}. \qquad (15)$$

Thus, it is useful to apply the following routine allowing essentially to improve convergence of process of iteration. From three consecutive approximations q_1, q_2 and q_3, make the expression:

$$q_4=\frac{q_1q_3-q_2^2}{q_1+q_3-2q_2}, \qquad (16)$$

where q_4 will appear essentially more precise than the previous approximations. With a view of improvement of convergence of process of iterations, formula (16) can be applied repeatedly.

4. Transition from the system of coordinates connected with the SV orbit to the system of a comet's orbit

In the system of coordinates connected with the SV orbit (if X-axis is directed to its perihelion) the coordinates of the SV, x_l, y_l, z_l, are expressed similarly to (3):

$$x_l=\frac{p_l\cos v_l}{1+e_l\cos v_l},$$

$$y_l=\frac{p_l\sin v_l}{1+e_l\cos v_l}, \qquad (17)$$

$$z_l=0.$$

The index l designates belonging to the system connected with SV orbit, and the value v_l for each separately taken moment t_l is determined from the solution of Kepler's equation:

$$tg\frac{v_l}{2}=\sqrt{\frac{1+e_l}{1-e_l}}tg\frac{E_l}{2}, \tag{18}$$

where

$$E_l - e_l \sin E_l = M_l. \tag{19}$$

The duration of flight of the SV is equal to

$$\Delta T = a_L^{3/2}(M_L - M_0), \tag{20}$$

where through M_0 and M_L are designated initial and final values of the mean anomaly of the SV, accordingly; a_L is the semimajor axis of the SV orbit.

For calculations of values of v_l by means of expressions (19) and (20), we shall divide interval ΔT into L equal parts.

Transition from coordinates x_l, y_l, z_l to coordinates x_0, y_0, z_0 in the system connected with an orbit of a comet, are made by the formulas of spherical trigonometry:

$$\begin{aligned}
x_0 &= x_l(\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i) - y_l(\sin\omega\cos\Omega + \cos\omega\sin\Omega\cos i),\\
y_0 &= x_l(\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i) - y_l(\sin\omega\sin\Omega - \cos\omega\cos\Omega\cos i),\\
z_0 &= \sin i(x_l\sin\omega + y_l\cos\omega),
\end{aligned} \tag{21}$$

where i, ω and Ω are the inclination, argument of perihelion, and the longitude of Node of the SV orbit relative to the comet's orbit. These values are calculated by means of expressions:

$$\begin{aligned}
\cos i &= \cos i_n \cos i_l + \sin i_n \sin i_l \cos(\Omega_n - \Omega_l),\\
\omega &= \omega_l - arctg\frac{\sin i_n \sin(\Omega_n - \Omega_l)}{-\cos i_n \sin i_l + \sin i_n \cos i_l \cos(\Omega_n - \Omega_l)},\\
\Omega &= arctg\frac{\sin i_l \sin(\Omega_n - \Omega_l)}{\sin i_n \cos i_l - \cos i_n \sin i_l \cos(\Omega_n - \Omega_l)} - \omega_n.
\end{aligned} \tag{22}$$

where the index n designates elements of an orbit *of n-th* comet relative to the ecliptic.

Finally, unnormalized probability of a collision of the SV with a meteor for all the time of flight is as follows:

$$P_{LN} = \frac{\Delta T}{L+1}\sum_{l=0}^{L}\sum_{n=1}^{N} e^{-\left(\frac{\Delta_{ln}}{\Delta_0}\right)^2}, \tag{23}$$

where N designates the total number of the comets' orbits used in the calculations.

We realize the software for calculation of the probability of a collision of a SV with meteors in FORTRAN. With a PC Pentium-4, at the presence of 500 elements of comet's orbits, and division of the interval ΔT into 1000 parts, this requires 4 s of Machine time.

Thus, the present research allows to choose, from several possible variants of a trajectory of interplanetary flight of the SV, the most safe, from the point of view of collisions with meteors.

5. References

1. On investigation on long-term orbital evolution of geostationary satellites, *Proc. 12th Intern. Sympos. "Space Flight Dynamics",* 1997, 53-57. R. I. Kiladze, A. S. Sochilina, K.V. Grigoriev, A. N. Vershkov.

2. On occasional changes of velocities of geostationary satellites, *US Nav. Obs., 20-23 Oct., 1998. Washington, DC,* 1999, 339-351. R. I. Kiladze, A. S. Sochilina, K. V. Grigoriev, A. N. Vershkov.

3. On statistics of changes in rates of drift among uncontrolled geostationary objects, *Proc. III European Conf. of Space Debris, ESOC*, 2001, 367-372. R. I. Kiladze, A. S. Sochilina, F. R. Hoots, R. France, A. N. Vershkov.

4. O novoy teorii dvijeniya geostacionarnih sputnikov. In: *Nasireddin Tusi i Sovremennaya Astronomia, Schemahinskaya AO* (About the new theory of motion of geostationary satellites, *In: Nasireddin Tusi and Modern Astronomy),* 2001, 113-121, R. I. Kiladze (in Russian).

5. O dinamicheskom metode kontrolya zasoreniya geostacionarnoy orbiti. *Trudi nauchnogo seminara "Ekologia i kosmos" 13 Fevralya 2004 g.*, *S.-Peterburg* (About a dynamic method of monitoring of a contamination of a geostationary orbit. *Proc. of a Scientific Seminar «Ecology and Space» on February, 13th, 2004)*, 2004, 19, A. S. Sochilina, R. I. Kiladze (in Russian).