On Some Difficulties of Planning the Search for a Space Object by Narrow-Angled (Narrow-Beam) Sensors

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When planning the search of a space object (SO) with the help of narrow-angled (narrow-beam) facilities based on rough a priori orbital information (that is in the case when the whole uncertainty domain of the SO ephemeris is not covered by one field of view of the sensor), an unforgivable fallacy usually takes place. The construction of the search plan (stretched in time) usually does not take into account the real structure transformation and deformation of the sought for SO current position uncertainty domain (CPUD) during its motion. Such negligence (neglect of real CPUD transformation) is followed by two kinds of errors in the constructed search plan. The first kind of error is connected with producing "slots" between the elements of a search plan, where one can lose the SO. The second kind of error means some redundancy of overlapping the search plan elements. The latter leads to ineffective expenditure of the search resources. To eliminate these errors a special search theory was developed and considered at our workshop[1, 2]. But in its frame the constructive methods were worked out only for the case (though practically very important) of a great positional error only in the argument of latitude [2, 3 4]. At the same time there exist many practically significant cases of large errors in different orbit elements, where due regard for the real structure transformation of CPUD is much harder. This is, by R. Bellman's words, "the damnation of dimensionality". And not only of dimensionality.

For the sake of simplicity, represent the SO CPUD as an uncertainty ellipsoid (UE) with the initial central position vector

$$R(t_0) = \begin{pmatrix} X(t_0) \\ Y(t_0) \\ Z(t_0) \\ \dot{X}(t_0) \\ \dot{Y}(t_0) \\ \dot{Z}(t_0) \end{pmatrix}$$

and the covariance matrix of errors

$$K_R(t_0) = \begin{Vmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} & \sigma_{XVx} & \sigma_{XVy} & \sigma_{XVz} \\ 0 & \sigma_Y^2 & \sigma_{YZ} & \sigma_{YVx} & \sigma_{YVy} & \sigma_{YVz} \\ 0 & 0 & \sigma_Z^2 & \sigma_{ZVx} & \sigma_{ZVy} & \sigma_{ZVz} \\ 0 & 0 & 0 & \sigma_{Vx}^2 & \sigma_{VxVy} & \sigma_{VxVz} \\ 0 & 0 & 0 & 0 & \sigma_{Vy}^2 & \sigma_{VyVz} \\ 0 & 0 & 0 & 0 & 0 & \sigma_{Vz}^2 \end{Vmatrix}.$$

Here, a method of exposing the "anatomy" of such a complicated process of the structural transformation of a 6-dimensional CPUD, during the SO motion relative to the observer, is proposed. This method allows constructively taking into account the very complicated real relative structural transformation of already observed CPUD points, and of those still not observed, for correctly "docking" the search plan elements.

For the sake of definiteness, define the picture plane (PP) as a plane coming through the center of UE normal to the main axis of sight (observer-target axis). One can see that its position changes with time. And one can hardly imagine what happens with the projections of all points of a 3-dimensional CPUD to this travelling PP, and especially what happens with relative positions of the projections of these points. The more so – with the projections of points of a 6-dimensional CPUD.

Suppose that we have a vector $R(t_0)$ and its covariance matrix $K_R(t_0)$, as the initial a priori metric data on the sought for SO. Let the projection of UE to PP at time t_0 look as it is presented at Fig.1, where $\beta\gamma$ is a reference frame in PP with the origin of coordinates in the given SO position at time t_0.

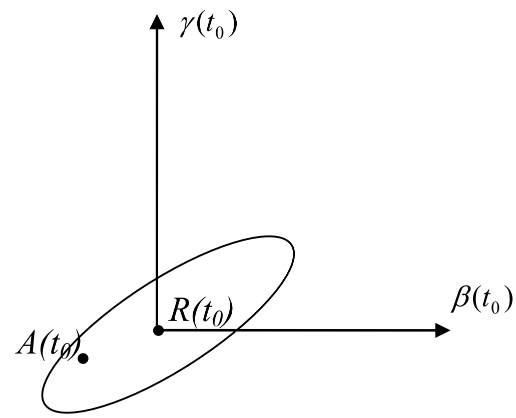

Fig. 1. The uncertainty ellipsoid projection in the picture plane

Pick out in this PP (at time t_0) within the UE projection an arbitrary point as if being sighted by the observer, $A(t_0) \equiv (\beta(t_0), \gamma(t_0))$.

In the 3-dimensional space XYZ (inertial reference frame) this point corresponds to the line segment $A_b(t_0) \equiv (X_b(t_0), Y_b(t_0), Z_b(t_0))$; $A_e(t_0) \equiv (X_e(t_0), Y_e(t_0), Z_e(t_0))$ normal to PP and limited by the UE surface. The beginning and the end of this line segment $A_b(t_0)$ and $A_e(t_0)$ have the phase coordinates

$$R_{A_b}(t_0) = \begin{pmatrix} X_{A_b}(t_0) \\ Y_{A_b}(t_0) \\ Z_{A_b}(t_0) \\ \{\dot{X}_{A_b}(t_0)\} \\ \{\dot{Y}_{A_b}(t_0)\} \\ \{\dot{Z}_{A_b}(t_0)\} \end{pmatrix} \quad \text{and} \quad R_{A_e}(t_0) = \begin{pmatrix} X_{A_e}(t_0) \\ Y_{A_e}(t_0) \\ Z_{A_e}(t_0) \\ \{\dot{X}_{A_e}(t_0)\} \\ \{\dot{Y}_{A_e}(t_0)\} \\ \{\dot{Z}_{A_e}(t_0)\} \end{pmatrix},$$

the first 3 coordinates (positional) of these vectors having definite values as the solution of the joint system of equations of the line $[A_b(t_0), A_e(t_0)]$ and the UE surface. The other 3 coordinates (velocities – in braces) are indefinite. The information on the limits of their indefiniteness (uncertainty) is contained in the covariance matrix $K_R(t_0)$ and should be extracted from the matrix. But it is not evident how to do this.

Here the means are proposed. Let us form two 3-dimensional subvectors

$$A_b^3(t_0)=\begin{pmatrix} X_{A_b}(t_0) \\ Y_{A_b}(t_0) \\ Z_{A_b}(t_0) \end{pmatrix} \quad \text{and} \quad A_e^3(t_0)=\begin{pmatrix} X_{A_e}(t_0) \\ Y_{A_e}(t_0) \\ Z_{A_e}(t_0) \end{pmatrix}$$

from 6-dimensional phase vectors $R_{Ab}(t_0)$ and $R_{Ae}(t_0)$, and name the former subvectors as pseudo measurements with zero-valued covariance matrices of errors. These subvectors are, so to say, the "deprojections" or "reprojections" of point A from PP back to 3-dimensional space.

Such a move is justified by the fact that the primarily selected point $A(t_0)$ in PP represents a possible (within the UE projection) position of the sought for SO in PP.

As the next step, we solve in consecutive order the program of refinement of the initial (6-dimensional) orbit vector $R(t_0)$, and its covariance matrix $K_R(t_0)$, using first the 3-dimensional pseudo measurement $A_b^3(t_0)$ and then $A_e^3(t_0)$. As a result, two refined (pseudo refined) vectors of phase coordinates corresponding to points $A_b(t_0)$ and $A_e(t_0)$ will be obtained:

$$R_{A_b}^r(t_0)=\begin{pmatrix} X_{A_b}(t_0) \\ Y_{A_b}(t_0) \\ Z_{A_b}(t_0) \\ \dot{X}_{A_b}^r(t_0) \\ \dot{Y}_{A_b}^r(t_0) \\ \dot{Z}_{A_b}^r(t_0) \end{pmatrix} \quad \text{and} \quad R_{A_e}^r(t_0)=\begin{pmatrix} X_{A_e}(t_0) \\ Y_{A_e}(t_0) \\ Z_{A_e}(t_0) \\ \dot{X}_{A_e}^r(t_0) \\ \dot{Y}_{A_e}^r(t_0) \\ \dot{Z}_{A_e}^r(t_0) \end{pmatrix},$$

and their covariance matrices $K^r_{Ab}(t_0)$ and $K^r_{Ae}(t_0)$. The components of the latter matrices are much smaller than those of the initial matrix $K_R(t_0)$.

Then, with the help of the model of the SO motion, we propagate vectors $R^r_{Ab}(t_0)$ and $R^r_{Ae}(t_0)$ to the time t_1 and obtain new vectors

$$R_{A_b}^r(t_1)=\begin{pmatrix} X_{A_b}(t_1) \\ Y_{A_b}(t_1) \\ Z_{A_b}(t_1) \\ \dot{X}_{A_b}^r(t_1) \\ \dot{Y}_{A_b}^r(t_1) \\ \dot{Z}_{A_b}^r(t_1) \end{pmatrix} \quad \text{and} \quad R_{A_e}^r(t_1)=\begin{pmatrix} X_{A_e}(t_1) \\ Y_{A_e}(t_1) \\ Z_{A_e}(t_1) \\ \dot{X}_{A_e}^r(t_1) \\ \dot{Y}_{A_e}^r(t_1) \\ \dot{Z}_{A_e}^r(t_1) \end{pmatrix}$$

and their covariance matrices $K^r_{Ab}(t_1)$ and $K^r_{Ae}(t_1)$.

After projecting these vectors to PP $\beta(t_1)\gamma(t_1)$ (at time t_1 it will be another PP), we obtain two points $A_b(t_1)$ and $A_e(t_1)$, that is the prediction mapping of points $A_b(t_0)$ and $A_e(t_0)$ to time t_1. To be more precise, these are only the centers of the uncertainty domains (the projection of "tiny" uncertainty ellipsoids) where points $A_b(t_0)$ and $A_e(t_0)$ are transferred when being predicted (propagated).

The question is: what is the whole image of the initially selected point $A(t_0)$ at time t_1? This situation is much harder, because so far we constructed only the ends of the uncertainty segment. Strictly, the same as we have handled with the ends of the uncertainty segment $[A_b(t_0), A_e(t_0)]$, we should handle with all its points. So, theoretically, we can obtain the whole image of point $A(t_0)$ at PP $\beta(t_1)\gamma(t_1)$. But this way is non-constructive. Practically, we can represent the segment $[A_b(t_0) , A_e(t_0)]$ by several points, and apply all the procedure to them. Eventually we approximately obtain the whole image of the point $A(t_0)$ at PP $\beta(t_1)\gamma(t_1)$ – see Fig. 2. At last we can envelop the entire domain.

So, we have clarified the appearance of the image (and how to get it) of an arbitrary selected point $A(t_0) \equiv (\beta(t_0), \gamma(t_0))$ in PP $\beta(t_0)\gamma(t_0)$ after the transfer from time t_0 to time t_1. The computer model of these transformations is under way now.

That was the central research problem because the more complicated figures (for example the telescope field of view in PP) can be handled and mapped by points.

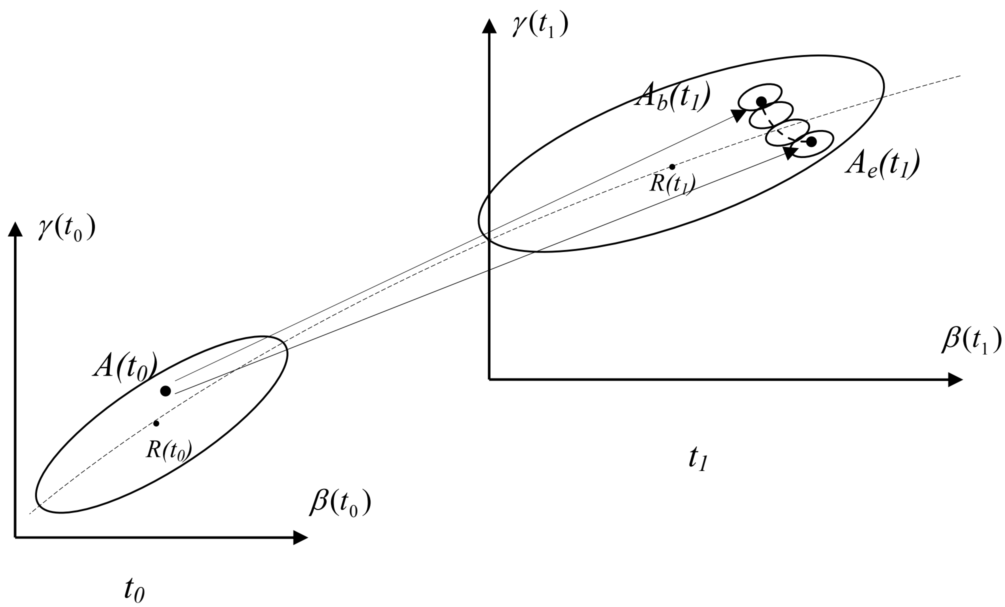

Fig. 2. The temporal transformation of the initially selected point $A(t_0)$ in PP (its mapping from time t_0 to time t_1).

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R E F E R E N C E S

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