The possibility of using non-coordinate measurements for evaluation of the parameters of the model of solar radiation pressure for the prediction of orbital motion

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Sometimes there are the difficulties connected with tracking and cataloging spacecraft fragments, which are on elliptic orbits, due to the orbital movement mispredictions of high area-to-mass space objects. Basically, it is caused by the incorrectness of the model of the solar radiation pressure influence on the orbit parameters. The translational motion change depends on the forces of the light pressure for the considered objects. These forces essentially vary when the orientation of the object, relative to the direction on the Sun, changes. In turn, the orientation of the object is caused by the influence of the moments of the same forces of light pressure. The estimation of the depth of such changes, and time of their displays, can be performed by an analysis of the non-coordinate optical (photometric) measurements, which are based on the character and parameter variations of the observable light curves.

Evolution of the movement of the simple model of the space object is considered in this paper. Influence of the moments of the light pressure forces on the change of the space object orientation is estimated from the simultaneous analysis of the photometric signal changes. The possibility of the application of non-coordinate measurements for the correction of the prediction algorithms of the orbital movement of the simple form of spacecraft fragments is shown.

1. Introduction

During the observations of high-space objects, having a large area to mass ratio, changes in the orbital parameters, that do not fit into the framework of the models commonly used in determining and predicting the orbits, are detected. It may be that in such calculations, in taking into account the light pressure force, the factor of light pressure is considered a constant. However, this may not be so, because this factor depends on the illuminated surface area, which depends on the orientation of the object. Thus, the motion of the object concerning the center of mass can affect its translational motion.

2. The moment of light pressure.

One of the kinds of objects with a large ratio of surface area to mass is an element of the destruction of a spacecraft. We assume that at the considered altitudes, major factors affecting the movement of such objects are the moments of light pressure forces (for more accurate calculations, in addition to these points, gravity must be taken into account). Significant changes in the rotational motions can cause the emergence of the so-called “propeller” moment, similar to that considered in [1] for the incident aerodynamic flow. The impact of the moment leads to a "turn" of the object.

To ease calculations, we assume the object is dynamically symmetric, consisting on each other's planes of two deployed water meter vanes (i.e. a model of a flat curved element, which may be a fragment of destruction). Under such assumptions just the propeller moment of light pressure will produce the maximum effect.

We shall describe the form of the object as follows. Let «blades of a revolving object » lay in one plane and make a rectangular. The specified rectangular is located in a coordinate plane Ox_1x_2 of the coordinate system $Ox_1x_2x_3$ (associated with the object coordinate system), formed by the principal central axes of inertia of the object, the point O is the center of the rectangle, and the axis Ox_1 passes through the middle of one of its sides. Blades enumerate the numbers 1, 2, attributing Number 1 blade, containing a segment of positive semi-axis Ox_1. Assume that the blades are rotated at the same angle in the same direction around its axis of symmetry.

Here are some of the geometric characteristics of such a propeller (fig. 1).

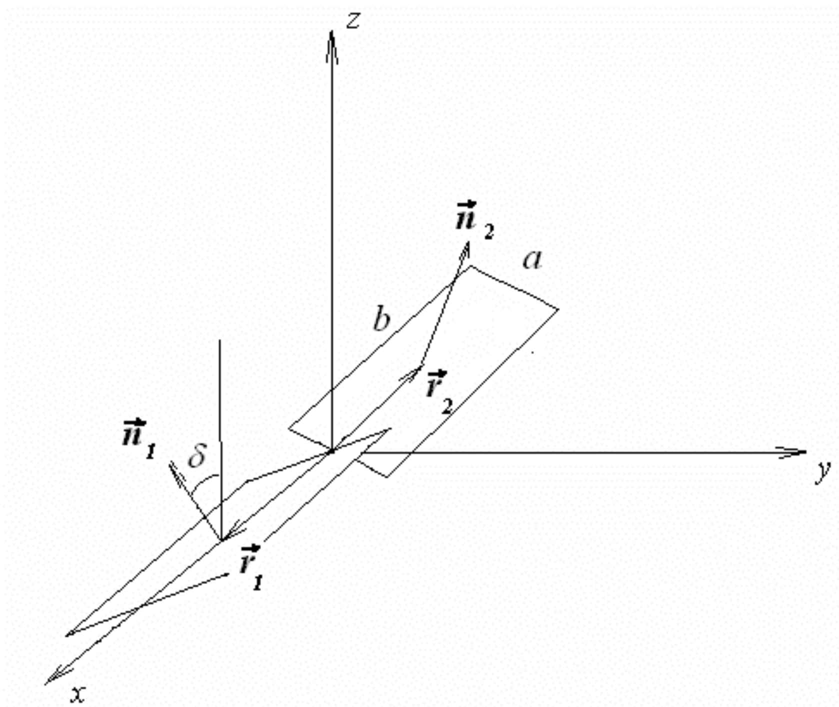

Fig.1. The object model

The normals to the blades are denoted n_i

$n_1 = (0, -\sin\delta, \cos\delta)$ $\quad\quad$ $n_2 = (0, \sin\delta, \cos\delta)$

Here, δ - angle of the blade about its longitudinal axis.

The radius vector of the geometric center of the i blade (the center of the rectangle) relative to the point O denoted $\mathbf{r}_i$ in the system $Ox_1x_2x_3$

$r_1 = (b/2, 0, 0)$ $\quad\quad$ $r_2 = (-b/2, 0, 0)$,

where a - the width of the blade, b - the length of the blade propeller.

Following [2] and by analogy with [1], we pass to the calculation of the main moment of light pressure forces acting on the satellite. We assume that the proportion, ε hitting on the blade, of photons reflected from the mirror, the proportion of photons, $1-\varepsilon$, absorbed completely. Mutual shading blades will not be taken into account. Under our assumptions, the main vector, representing the blade forces of light pressure, is determined by formulas

$$\mathbf{F}_s = \varepsilon \mathbf{F}_s^{(1)} + (1-\varepsilon)\mathbf{F}_s^{(0)},$$

$$\mathbf{F}_s^{(1)} = -2p_s\sigma_s\sum_{i=1}^{2}(\mathbf{s}\cdot\mathbf{n}_i)^2\mathbf{n}_i, \quad \mathbf{F}_s^{(0)} = -p_s\sigma_s\mathbf{s}\sum_{i=1}^{2}(\mathbf{s}\cdot\mathbf{n}_i).$$

Here, $p_s \approx 4.64\cdot 10^{-6}\text{N / m}^2$ - light pressure on the ideal flat mirror perpendicular to the Sun's rays on the heliocentric orbit of the Earth, σ_s - the area of one blade. The sums in the expressions for $\mathbf{F}_s^{(0)}$ and $\mathbf{F}_s^{(1)}$ are calculated in scalar form in $Ox_1x_2x_3$ the coordinate system. $\mathbf{s}$ is adopted as the direction "object - the Sun."

The main moment of the forces of light pressure applied to object is determined by formulas

$$\mathbf{M}_s = \varepsilon\mathbf{M}_s^{(1)} + (1-\varepsilon)\mathbf{M}_s^{(0)},$$

$$\mathbf{M}_s^{(1)} = 2p_s\sigma_s\sum_{i=1}^{2}(\mathbf{s}\cdot\mathbf{n}_i)^2(\mathbf{n}_i\times\mathbf{r}_i), \quad \mathbf{M}_s^{(0)} = p_s\sigma_s\mathbf{s}\times\sum_{i=1}^{2}(\mathbf{s}\cdot\mathbf{n}_i)\mathbf{r}_i.$$

As the pole are taken point O. The written out sums were calculated in a scalar form in the $Ox_1x_2x_3$ coordinate system.

These formulas refer to the sunlit orbit site of the cosmic object. In the shadow of the Earth the light pressure force moment acting on an object is zero. At this stage this shading is not considered.

After simple, but cumbersome, calculations we obtain

$$\mathbf{F}_s^{(1)} = -2p_s\sigma_s\left\{4(\mathbf{s}\cdot\mathbf{e}_y)((\mathbf{s}\cdot\mathbf{e}_z)\sin^2\delta\cos\delta\mathbf{e}_y + \left[\sin^2\delta(\mathbf{s}\cdot\mathbf{e}_y)^2 + \cos^2\delta(\mathbf{s}\cdot\mathbf{e}_z)^2\right]\cos\delta\mathbf{e}_z\right\}$$

$$\mathbf{F}_s^{(0)} = -p_s\sigma_s\mathbf{s}2\cos\delta(\mathbf{s}\cdot\mathbf{e}_z)$$

And for the moments of forces -

$$\mathbf{M}_s^{(1)} = b/2p_s\sigma_s\left\{\left[\sin^2\delta(\mathbf{s}\cdot\mathbf{e}_y)^2 + \cos^2\delta(\mathbf{s}\cdot\mathbf{e}_z)^2\right]\sin\delta\mathbf{e}_z - 4(\mathbf{s}\cdot\mathbf{e}_y)((\mathbf{s}\cdot\mathbf{e}_z)\sin\delta\cos^2\delta\mathbf{e}_y\right\}$$

$$\mathbf{M}_s^{(0)} = -b/2p_s\sigma_s\left[\mathbf{s}\times\mathbf{e}_x\right]\sin\delta(\mathbf{s}\cdot\mathbf{e}_y)$$

Here $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ - are the related coordinate system.

3. The equations of rotational motion of the object.

The satellite is solid. Its rotational motion equation, due to a moment of light pressure, can be written as

$$\frac{dL_X}{dt} = M_X \qquad \frac{dL_Y}{dt} = M_Y \qquad \frac{dL_Z}{dt} = M_Z$$

Here L - the kinetic moment of the satellite in its motion relative to the center of mass (L_X, L_Y, L_Y.-- its components). M_X, M_Y, M_Z -- components of the total moment of light pressure forces on the axes of the "solar" coordinate system, which we introduce as follows: O - center in the center of mass of the object, OX, OZ - in the ecliptic plane, OX is parallel to the direction of the Sun - vernal equinox point, OY - completes the system to the right. For the variables by which to describe the motion, we choose the following set

L, ρ, σ, ϑ, ψ, φ

Where L - the module of a kinetic moment vector,

ρ - an angle between a vector of the kinetic moment and axis OY of "solar" coordinate system,

σ - an angle between projection L to plane XOZ and axis OX,

ϑ - a nutation angle,

ψ - a precession angle,

φ - an angle of self-rotation.

Then the full system of the equations, describing the movement of an object, will be written down as [2]:

$$\dot{L} = (M_X \sin\sigma + M_Z \cos\sigma)\sin\rho + M_Y \cos\rho$$

$$\rho = \frac{1}{L}\left[(M_X \sin\sigma + M_Z \cos\sigma)\cos\rho - M_Y \sin\rho\right]$$

$$\sigma = \frac{1}{L\sin\rho}(M_X \cos\sigma - M_Z \sin\sigma)$$

$$\dot{\psi} = L\left(\frac{\sin^2\varphi}{A} + \frac{\cos^2\varphi}{B}\right) + \frac{1}{L}\{-M_X[ctg\vartheta(\cos\sigma\sin\psi + \sin\sigma\cos\rho\cos\psi) + ctg\rho\cos\sigma] +$$
$$+M_Y \sin\rho\, ctg\vartheta\cos\psi + M_Z\left[ctg\vartheta(\sin\sigma\sin\psi - \cos\sigma\cos\rho\cos\psi) + ctg\rho\sin\sigma\right]\}$$

$$\dot{\varphi} = L\cos\vartheta\left(\frac{1}{C} - \frac{\sin^2\varphi}{A} - \frac{\cos^2\varphi}{B}\right) + \frac{1}{L\sin\vartheta}\{M_X(\sin\sigma\cos\rho\cos\psi$$
$$+\cos\sigma\sin\psi) - M_Y \sin\rho\cos\psi + M_Z(\cos\sigma\cos\rho\cos\psi - \sin\sigma\sin\psi)\}$$

$$\dot{\vartheta} = L\sin\vartheta\sin\varphi\cos\varphi\left(\frac{1}{A} - \frac{1}{B}\right) + \frac{1}{L}[-M_X(\sin\sigma\cos\rho\cos\psi\, ctg\vartheta + \cos\sigma\, ctg\rho + \cos\sigma\sin\psi\, ctg\vartheta) +$$
$$+M_Y \sin\rho\cos\varphi\, ctg\vartheta + M_Z(-\cos\sigma\cos\rho\cos\psi\, ctg\vartheta + \sin\sigma\, ctg\rho + \sin\sigma\sin\psi\, ctg\vartheta)]$$

A, B, C - moments of inertia of the object relative to the axes Ox_1, Ox_2 and Ox_3, respectively.

The equations allow for further simplification. Namely, if during one turn of the satellite in an orbit its own kinetic remains almost unchanged, then the equation can be averaged over the orbital motion [2]. In averaging, the orbital motion of the satellite should be considered as a Keplerian elliptic, whose vector L and **s** - are constant.

Substituting in the equations expressions for the moments of the forces of light pressure, and solving them, it is possible to estimate the evolution of the space object rotation under the influence of the propeller moment. The following figures show the results of calculations of changes in the parameters of rotation within the proposed model. The initial rotation period of the object is about 20 sec.

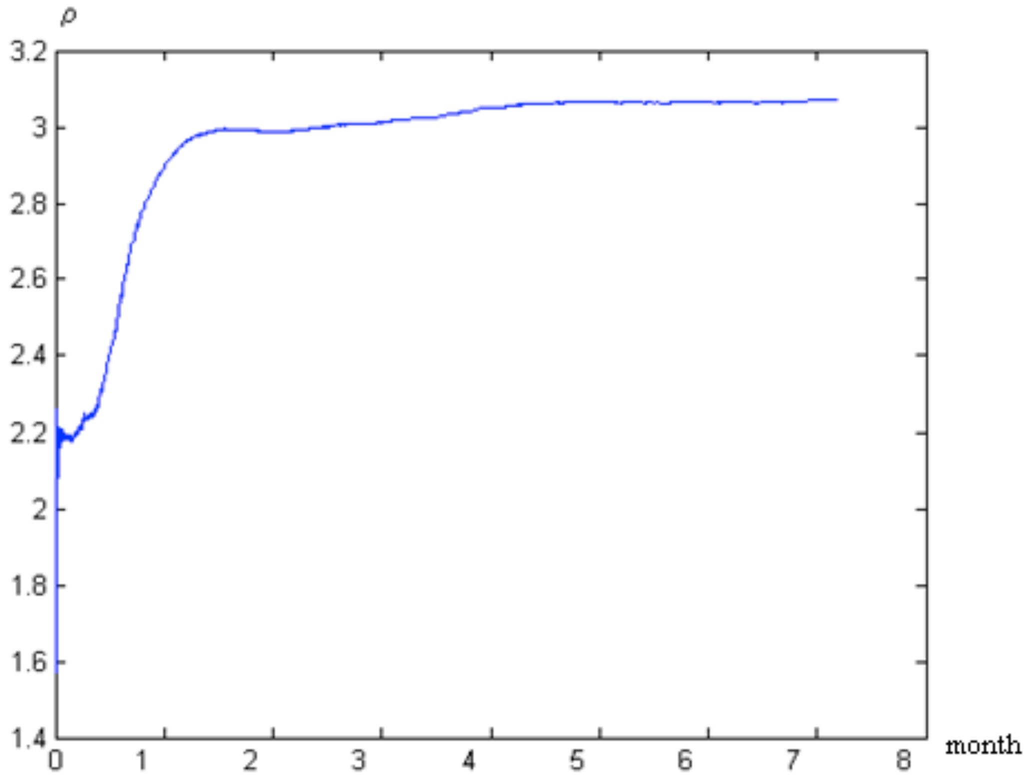

Fig. 2. Angle ρ orientations of the vector of the kinetic moment

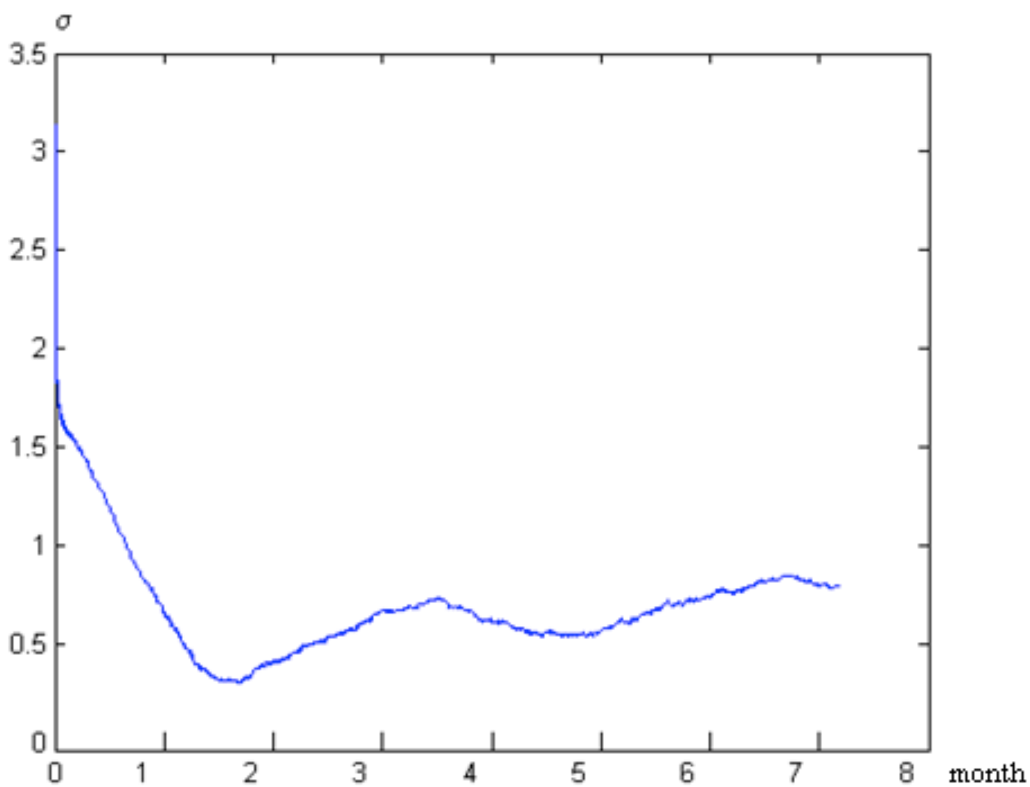

Fig. 3. Angle σ orientations of the vector of the kinetic moment

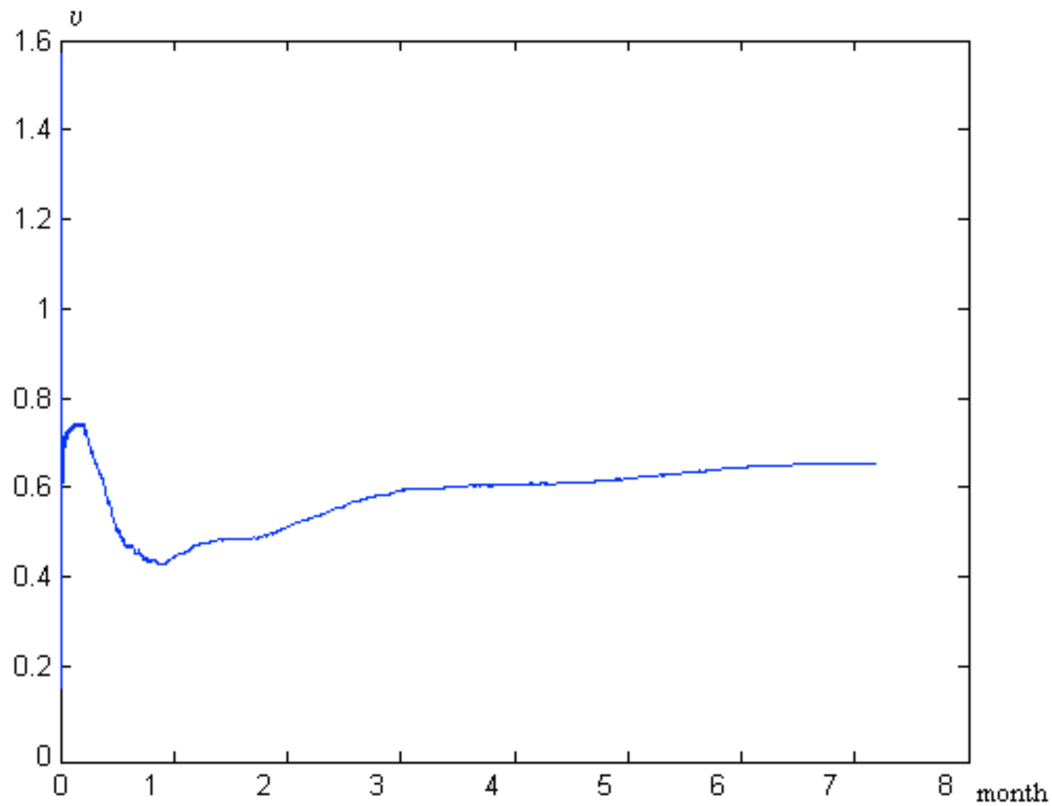

Fig. 4. Nutation angle ϑ

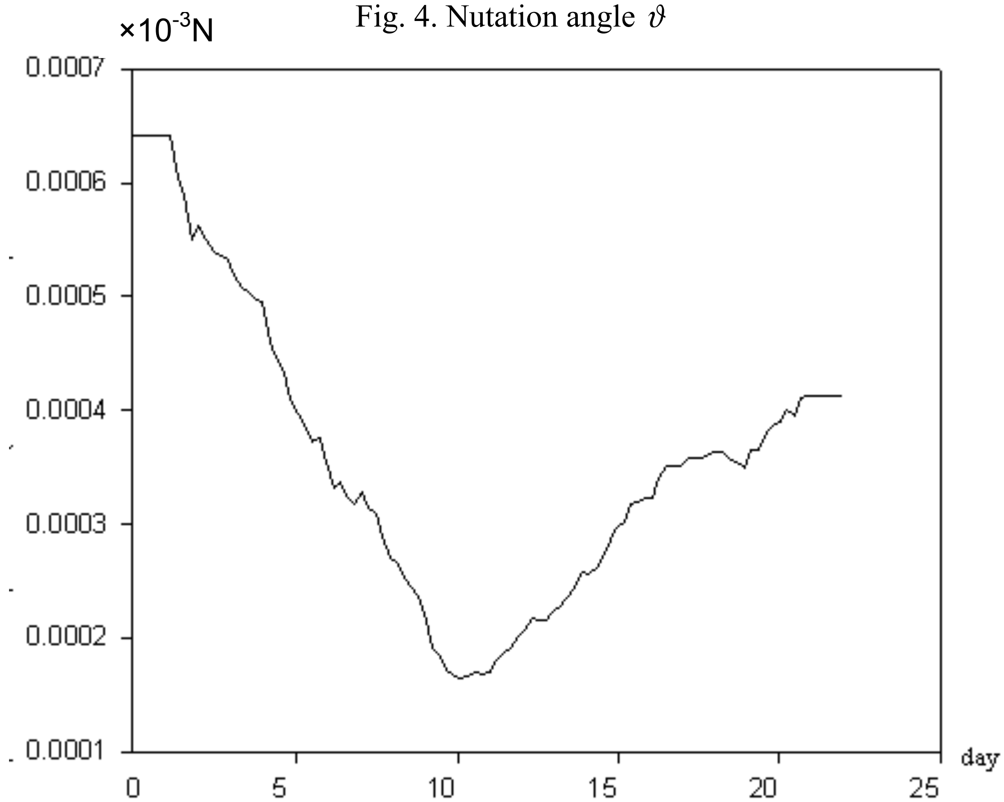

Fig. 5. The average component of force in radial direction

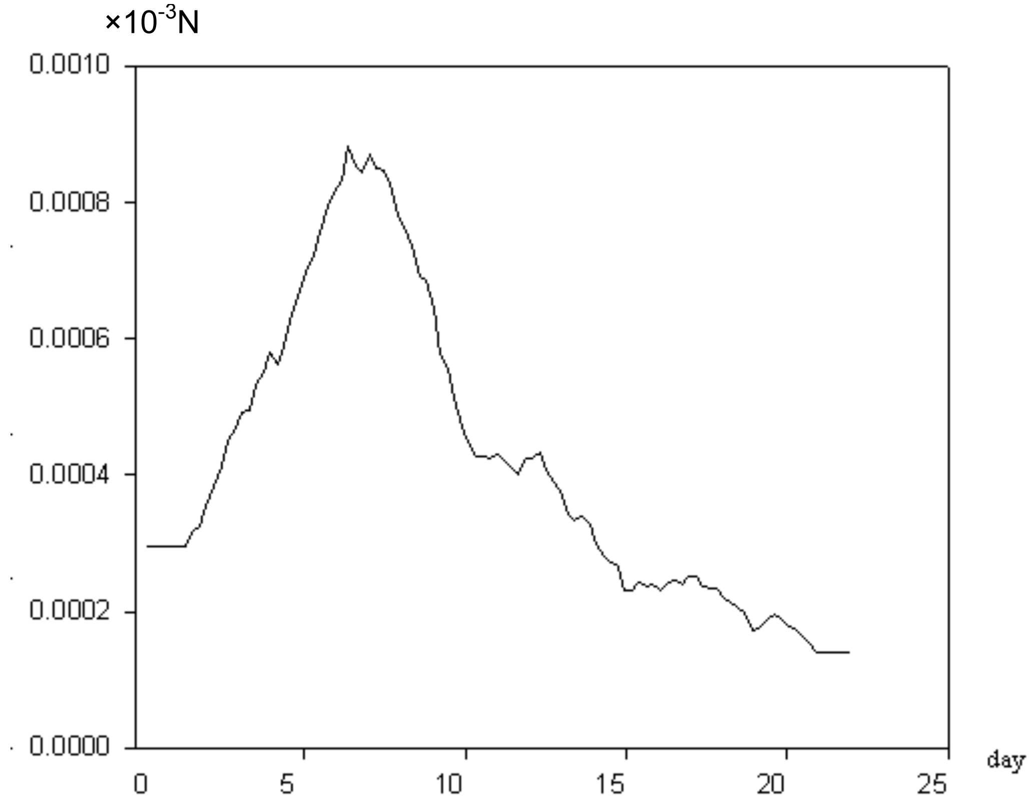

Fig. 6. The average component of force in tangential direction

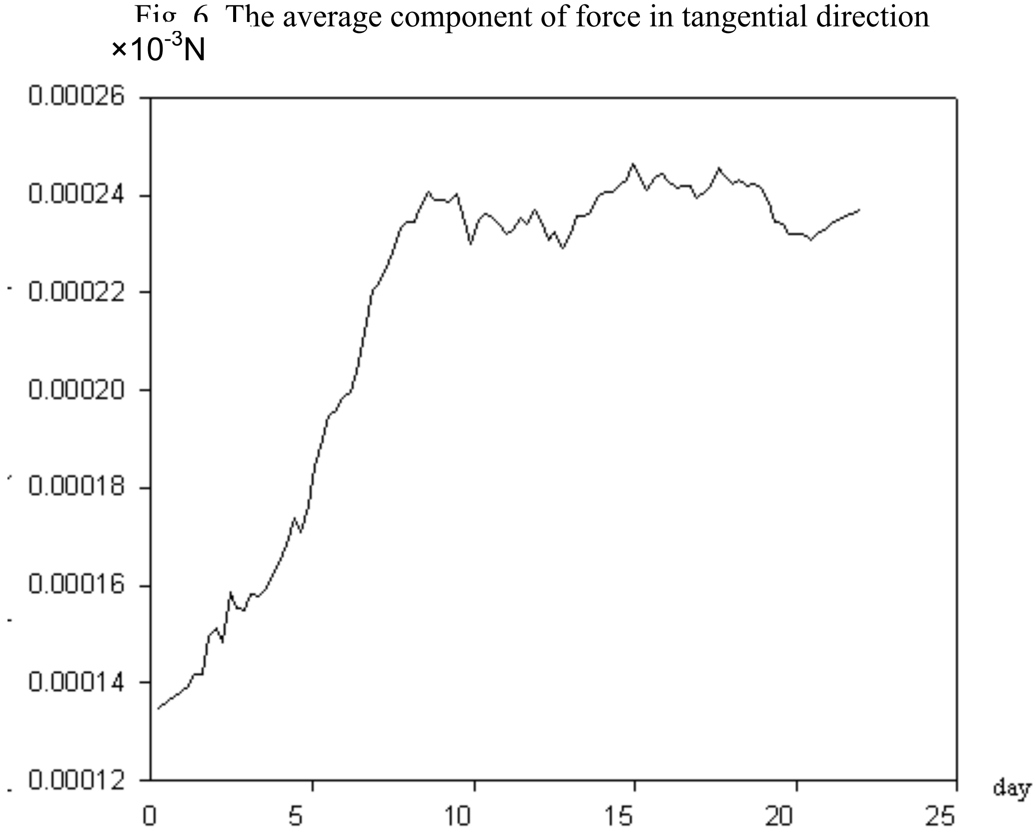

Fig. 7. The average component of force in direction perpendicular planes ecliptic

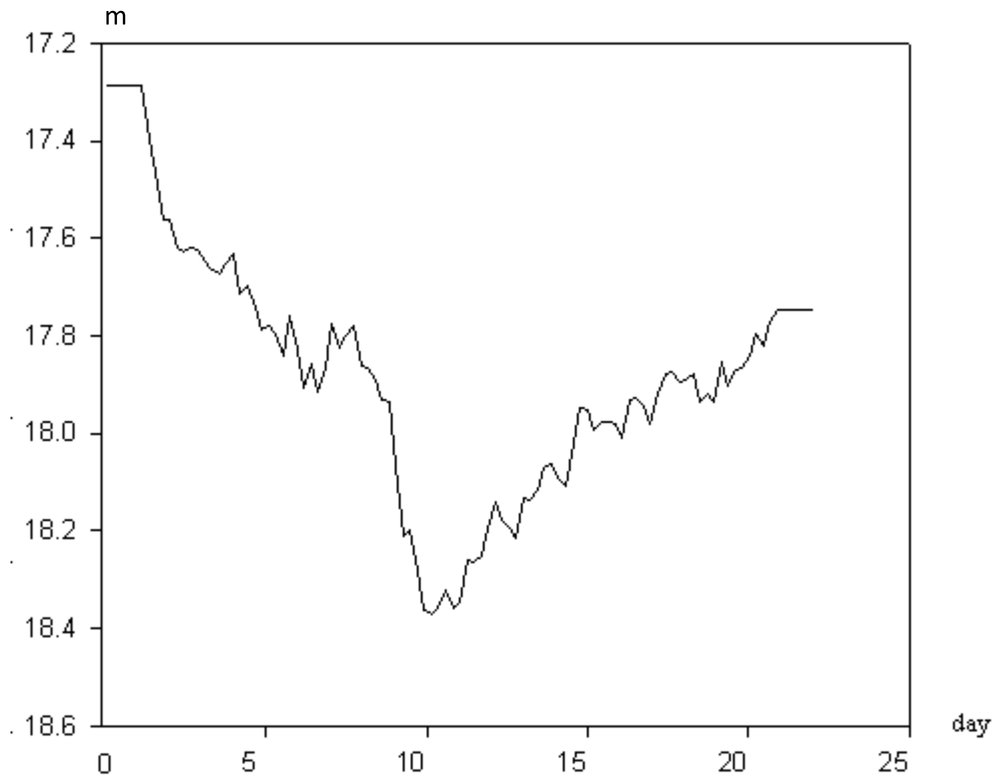

Fig. 8. The average brightness over the interval of averaging force

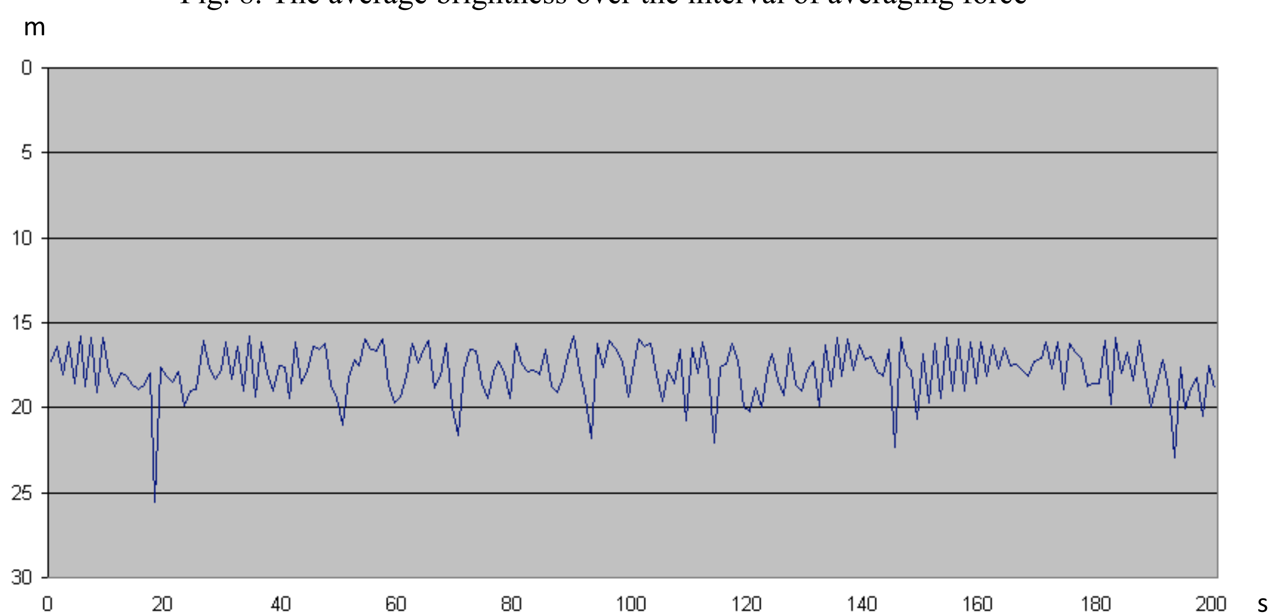

Fig. 9. Behavior of brightness in photometric realization in the beginning of the interval of modeling (mirror component)

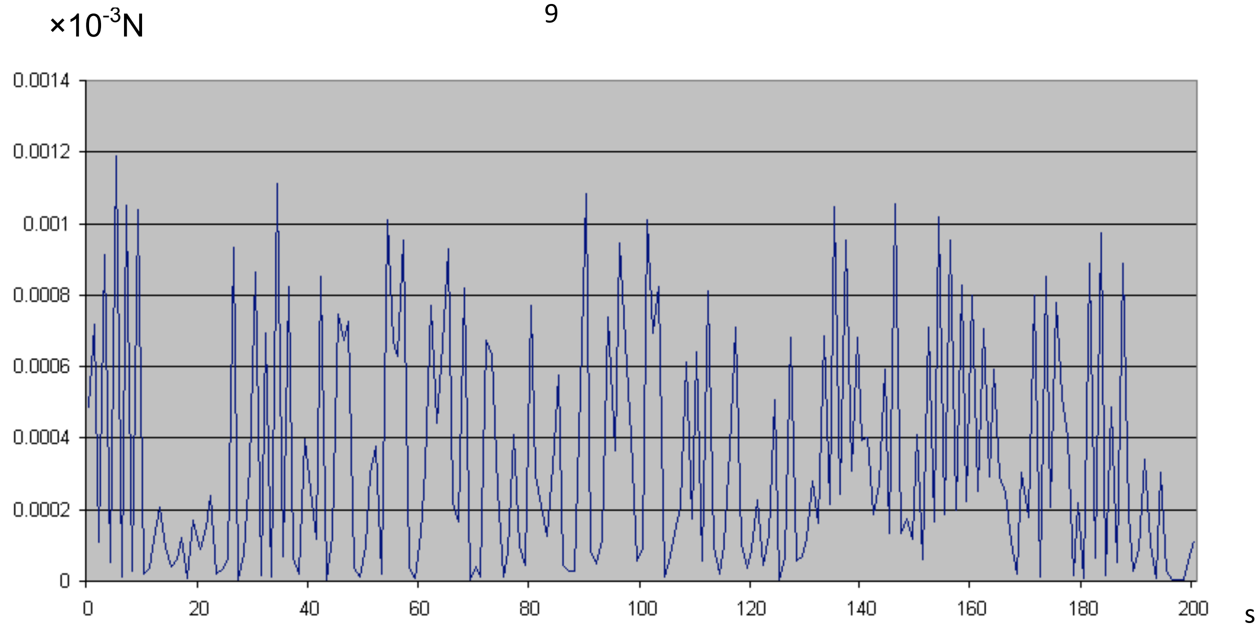

Fig. 10. The component of force in radial direction corresponding to the photometric realization in the beginning of the interval of modeling (fig. 9)

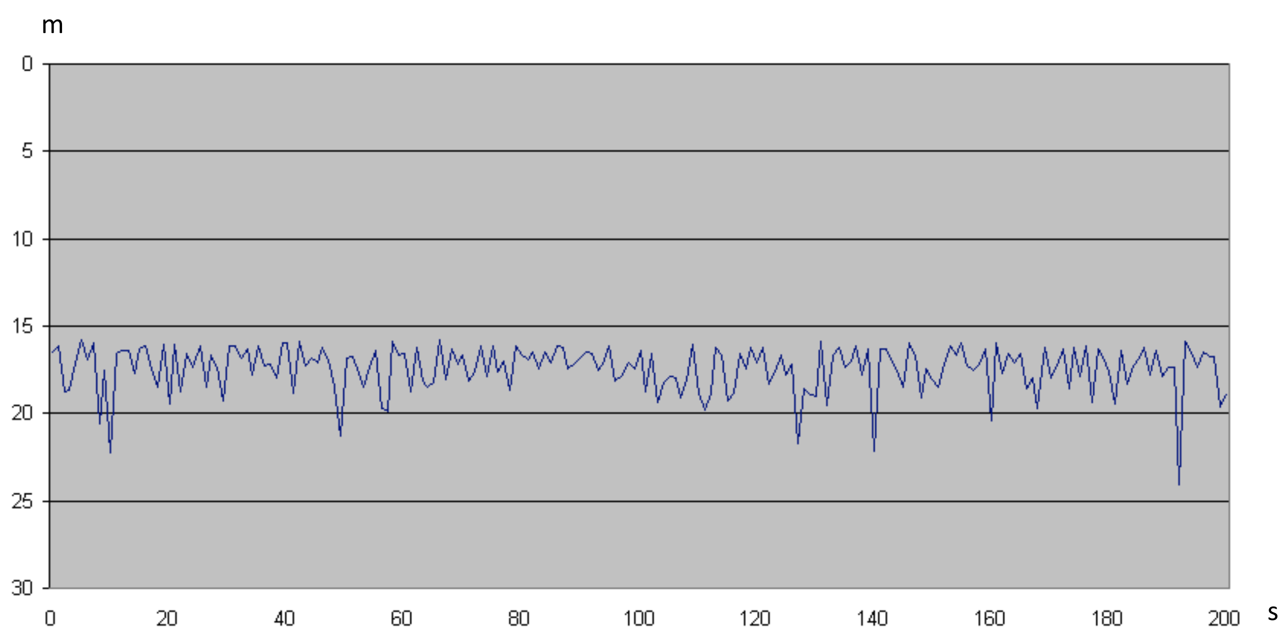

Fig. 11. Behavior of brightness in photometric realization at the end of the interval of modeling (mirror component)

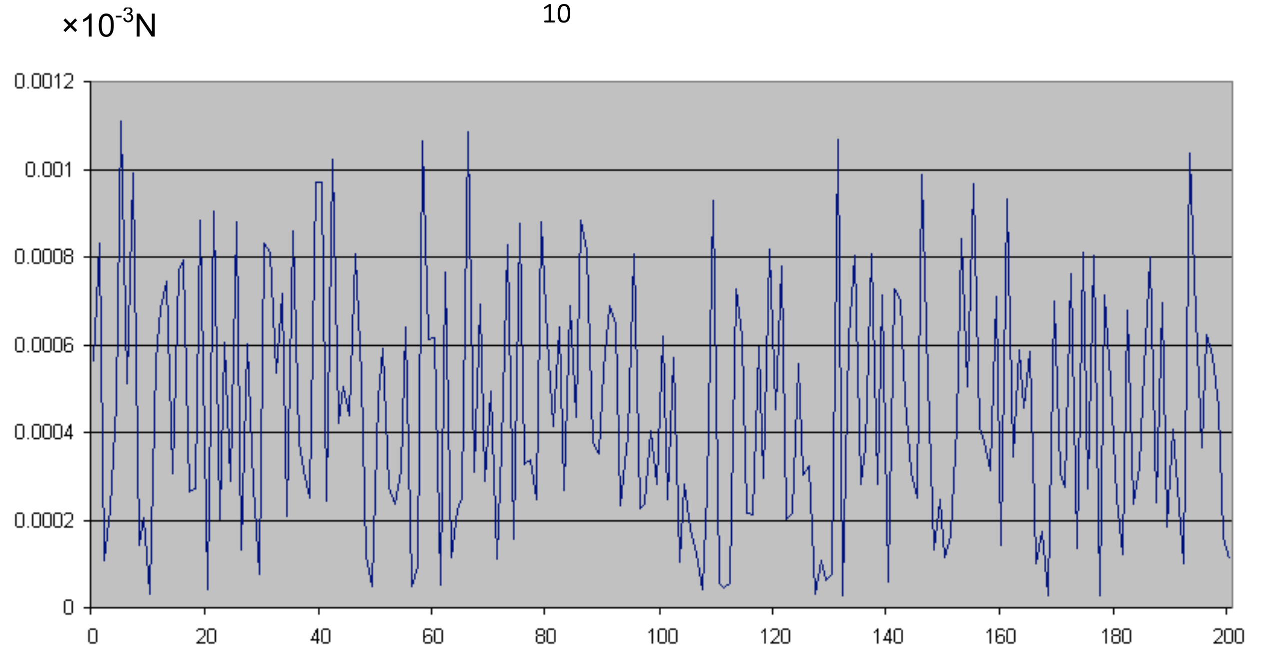

Fig. 12. The component of force in radial direction corresponding to the photometric realization at the end of the interval of modeling (fig. 11)

Thus, the motion of the center of mass can significantly change under the action of the pressure of sunlight. In line with these changes, and changes of the area of the object illuminated by the Sun, and, hence, the character of the photometric measurements of the space object, as well as the forces of solar radiation pressure, the translational motion is affected.

As can be seen from a comparison of fig. 5 and 8, variations of the radial component of force acting on an object are rather well correlated with the changes in the average brightness. The correlation coefficients, between changes in the light function and the values of the forces acting on an object in the radial direction at the beginning of the interval modeling, are 0.989, and 0.992 in the part at the end of this interval (correspondingly the graphs in Figures 9 - 10 and 11 - 12).

- Thus, the motion relative to the center of mass can significantly change under the action of the solar radiation pressure. Along with these changes the area of the satellite illuminated by the Sun varies, and, thus, the features of the photometric measures. The solar radiation pressure forces, affecting the motion of the satellite, vary respectively.
- For further research and validation of these results, we should perform the observations and tracking of satellites affected by solar radiation pressure, along with acquisition of their photometric characteristics.

References

1. Beletsky V.V. Essays about movement of space bodies. M., "Science", 1977.
2. Beletsky V.V. Movement of the artificial satellite relative to the center of mass. M., "Science", 1965.