Evaluating Gooding Angles-only Orbit Determination of Space Based Space Surveillance Measurements

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ABSTRACT

Geosynchronous satellites have traditionally been some of the least accurately maintained orbits with existing ground based sensors for several reasons. MIT/LL successfully operated the MSX/SBV sensor to provide additional observations to AFSPC for processing of GEO space objects for many years. Resulting GEO orbits witnessed a large improvement in accuracy. There are several US Air Force initiatives to introduce future Space Based Space Surveillance (SBSS) systems into the US AF Space Surveillance Network (SSN) sensor mix, but to date, none are operational. This paper reexamines expected orbit determination results for SBSS observations starting with the Initial Orbit Determination (IOD) of the observations, the placement of the SBSS sensor orbit, and data fusion with ground based radar and optical measurements. The goal is to investigate expected IOD and OD techniques to process the observations to improve the GEO Space Situational Awareness (SSA). In each case, simulated data is used to show the expected results and uncertainties of the resulting orbits. Analytical Graphics Inc. Orbit Determination Toolkit's (ODTK) implementation of the Gooding technique of IOD is used to illustrate performance for routine and near-singular observation geometries.

1. Introduction

A precise definition for Space Situational Awareness (SSA) is difficult to find, but for this paper, let's define SSA as the process by which an organization maintains a catalog of all objects in space, to some level of accuracy, in a timely fashion, and with the knowledge about the particular missions. Several relevant attributes are discussed in Vallado (2007, pg 831): complete and robust, timely and efficient, standardized and maintainable, accurate, and importantly, trusted. Many of these will show up in various locations through this paper.

Accurate cataloging of Geosynchronous (GEO) satellites is difficult for many reasons. Optical sensors provide angles-only observations that are easily limited by poor weather, and without range, the orbit determination lacks a critical component. Deep space radars are limited to just a couple of sites. Some sensors experience coverage gaps where satellites can be missed. Lost satellites occur often, primarily because of the inability to process through maneuvers. There is little surveillance, as opposed to the tracking of known objects. Small objects are difficult to observe from the ground (say 10 cm or less). There are observability problems where the sensor has limited or no relative motion with respect to the satellite. The correlation problem with GEO observations is compounded by satellites that maneuver often (sometimes daily) to keep in a certain "box" (near continual East-West, North-South maneuvering). Satellites are often co-located in popular locations for downlink or coverage. Third body perturbations affect the orbits with long-term inclination changes – both for debris and operational satellites. Solar Radiation pressure effects are difficult to model and prediction is not very accurate.

In January 2001, the Space Based Space Surveillance (SBSS) project launched an effort to develop satellite based-observations to improve SSA capabilities for AF Space Command. The constellation of space based sensors would observe primarily GEO satellites to add observations in regions where ground based sensors have limited observability, or are prevented from observing due to weather or other reasons.

The first SBSS test occurred with the Advanced Concept Technology Demonstration (ACTD) Mid-Course Space Experiment/Space Based Visible (MSX/SBV) sensor (Sharma et al, 2002). Launched in 1996, the satellites original mission payloads had mostly failed by 2002, but the SBV sensor remained active for several years, during which the satellite provided space based observations to the AFSPC satellite cataloging function. A Block 10 satellite was contracted in 2004 to bridge the gap from when MSX/SBV failed to the beginning of operations of the follow-on SBSS satellites. By 2005, independent review teams found serious problems with the Block 10 program. In 2006, the program was restructured, costs rose, and a new schedule was output. To date, the satellite still has not been launched.

Given the long program delays and uncertain future, a quick re-look at the expected accuracies from angles-only IOD techniques and subsequent Orbit Determination from different orbital locations seems prudent. Sharma et al. (2002) provides a good summary of the LEO experience with MSX/SBV and some of those sensor parameters are used for this study. The goal of this paper is to explore how additional SBSS observations could improve GEO SSA and to look at some options for placement of the sensors. The sensor placements in this paper could be as simple as hosted payloads on other satellites (ie GPS) depending on mission requirements, or a fully dedicated satellite in a different orbit.

2. Background – Angles-only techniques

The purpose of Initial Orbit Determination (IOD), especially in the case of angles-only data, is to obtain an initial estimate that is close enough to the true orbit to enable subsequent Least Squares (LS) or Kalman Filters (KF) processing to be successful. Thus, differences from a known orbit may be present, but the LS and KF processes can often arrive at a converged solution – a situation that indicates success. In cases where the IOD method does not yield a sufficiently accurate answer to obtain convergence in the LS or KF processing, those cases fail the IOD objective.

Angles-only techniques are most commonly used with optical sensors and their observations of geosynchronous satellites. Lacking range information, angles-only techniques are inherently less robust than techniques that also process range, and optionally rate information. As such, one would not expect to obtain a perfect (2-body) solution, even given perfect observational data, unless you happen to guess the exact initial range estimate. Even in this latter case, the technique may not provide the precise sought after result. Thus, we have choices to make in selecting the best approach to use angles-only IOD results and how to process them to provide a sufficiently accurate solution to a subsequent estimation process. Considerable attention is paid in the literature addressing techniques to help improve the initial estimates from angles-only methods (Moulton (1914, Long et al 1989, etc.). This paper takes a simpler approach using the processing power of the modern computer, and the availability of advanced techniques in tools like Orbit Determination Toolkit's (ODTK) to ingest the data and quickly output numerous trial results. Keep in mind that the processing described in this paper is not intended to replace the subsequent processing via LS and KF.

Observability is also very important. If the observer and target are coplanar, then the orbit plane of the target is essentially unobservable from the observer. For GEO to GEO processing you need at least 1 degree of planar separation to have the target orbital plane observable from the sensor. The results may be slightly different if the sensor is extremely accurate, but an exactly coplanar tracker and satellite cannot work. It's pure geometry and math, and nothing more. Analysis of the resulting simulations in this regime are not meaningful.

Gooding angles-only is one of a few popular methods for angles-only orbit determination IOD (Vallado 2007, Ch 7). While we know that a single instance of an angles-only technique will not produce an exact solution, it's sometimes problematic to determine which observation points to use when attempting a problem. There are several variables that are important to vary and take into account when testing the success of an IOD method. These are approximately listed in order of importance to the final solution.

a. The **spacing and selection of the observations** is important. The time to process a CCD image and determine start and end points imposes a reasonable estimate for the closest spacing for optical observations of about 30 seconds. For GEO satellites, these "can" be relatively close (in angular separation). In the Gibbs and Herrick-Gibbs approaches, widely spaced and closely spaced (respectively) observations are processed with greater fidelity. The same principle applies to angles-only techniques. Thus, for a given set of observations, we can have a number of data approaches to select inputs to the IOD method. Depending on the number of observations, we could process all the sequential groups of data (1-2-3, 2-3-4, ..., 7-8-9, 8-9-10, etc.) and we will find a large number of similar results, and a few outliers in the individual solutions. We could also look at the total permutations of all available combinations of the data. This can produce quite a large number of samples as the number of observations increase. Be careful with the results – data from observation points 1-2-3 and 178-179-180 will be for different locations (and epochs) in the orbit. The true anomaly (or appropriate equivalent orbital element – a fast variable), and will not be comparable between the two trials. Other orbital elements may need correction for 0-360 deg switching (values of right ascension of the node of 357 and 2 degrees for example). We could use the median or mode values of the orbital elements of the various solutions, of the possible combinations. A least squares approach could even be of benefit here. One caution to consider when assembling the various combinations is the possibility of using observations from two different passes (a pass here being defined as the time interval a satellite is visible to the sensor). It's possible to have observations from multiple passes and this may or may not produce realistic results.
b. **Multiple satellite processing**. The results are improved significantly if multiple satellites are estimated in the solution. This is due to the fact that the estimation can better model the sensor statistics and any correlations.

c. The **observability** introduced by the geometry of the sensor and satellite introduces limitations in the accuracy. Essentially, certain geometries diminish the amount of relative motion between the 2 satellites, so some satellites may be visible, but just not have sufficient relative motion to obtain reliable orbits from a single sensor. This is especially true for satellites in the same orbit, but ahead or behind the SBSS.
d. **Accuracy of the host sensor platform**. The SBSS must be in a well known orbit, or at least an orbit which has good tracking information. For LEO and GPS type orbits, this is relatively easy to accomplish with existing radar, transponder, GPS, etc data sources. For GEO, it becomes more challenging, unless there is some form of transponder, or perhaps a GPS receiver to provide more accurate tracking information. Relying (in GEO) on optical angles-only techniques places an undue burden on the analysts tasked with maintaining that satellite orbit, and can easily degrade the resulting accuracy of the SB measurements collected.
e. The **initial estimate of the range** is important, and one that is discussed at length in the literature because it is such a key to the overall success or failure of angles-only techniques. For the case of satellite observations, we can usually put a reasonable estimate (bounds) on the values that would be required. However, the precision of the answer is directly related to the accuracy of the initial guess.

3. Outline of Test Cases and Process

Extensive scripting was required to setup the various scenarios, but a methodology is in place to do the following tasks.

- Produce identical satellites and facilities in ODTK and STK, including constraints on sensors, facilities, etc. This is necessary to obtain realistic times of simulated measurements in ODTK because the access (visibility) calculations are handled in STK, but the times are set in the simulator in ODTK.
- Limit continuous tracking intervals. When dealing with GEO satellites (especially), there are many sensors, both ground and space based, that have full coverage 24 hours a day of an RSO. While this could be an operational mode, it's less than realistic in many ways. Thus, a script was developed to limit the tracks of data to both a ground and space based sensor to some time (say 10 or 20 min), and to repeat the process of placing a track within a larger window every so often (say every 360 min).
- Script a method to perform many trials automatically. This was especially true for the IOD tests which required numerous evaluations, changing only one or two items between runs. It was also useful for testing the various OD runs using different pass lengths and cycle times.

For the IOD, we can examine orbital elements or cartesian position and velocity vectors. Because the goal is to arrive at the best possible solution to start a batch LS or KF OD process, it's desirable to arrive at an answer that is valid near the starting point of the data to minimize any propagation errors at the start of the OD process. Orbital elements represent a better option because over a "short" interval of time (a rev or so), they do not change significantly and can be averaged. State vectors would require propagation to a common time before averaging and depending on the intensity of the perturbation forces, this could introduce significant error into the solution. Thus, the focus here is exclusively on orbital elements, specifically semimajor axis, eccentricity, inclination, right-ascension of the node, argument of perigee, and true anomaly / argument of latitude.

We'll examine the following broad processing approaches; single point solutions, sequential sequences of observations (123, 234, 345, …), and permutations of all combinations of observations (123, 234, 134, 358, …). This is done for 3 SBSS potential orbits; LEO (MSX-SBV-like), MEO (GPS-like), and GEO.

The simulator is a key to getting accurate results for analyzing the various configurations. We simulate the SBSS observational data every 30 seconds (to approximate the frequency of observation generation from an optical sensor). The ground sensors generate observations about every 10 seconds, again to be somewhat consistent with real-world operations. For the simulator, the degree of error that the simulator introduces must be realistic. In ODTK, this is set in the ErrorModeling section of the simulator. You can turn all the error sources off, or specify certain errors to be included. You can also specify the relative magnitude of the error sources (1.0 being the default). The simulator uses the initial values from the satellites in the ODTK setup, so if you've made changes to the satellites (possibly by trials of the IOD and transferring the results to the satellites), the simulated observations could be potentially off from the values you would expect. There are many options to introduce error into the simulation. The two predominant factors in the all error sources for our cases were white noise (which simply introduces Gaussian errors into the simulation to "fuzz" the data up), and orbital errors (which deviate the orbital positions).

The sensor used characteristics (FOV, etc.) from the SBV sensor. The sensor was not constrained to look in a particular direction. Thus, these results should be considered slightly optimistic because some of the sensor engagements may not have been possible under a fixed sensor pointing assumption. Future study may examine fixing the sensor in a particular direction, although this may only be reasonable for an SBSS located in GEO orbit.

4. Test Case Setup and Initial Parameters

For the first section of the paper, there were no tracking observations for the SBSS satellite so the SBSS satellite used a reference ephemeris (from a filter-smoother run) and was not estimated. This was necessary to isolate the IOD process without injecting too many variables at once. To ensure the orbit was the same between STK and ODTK, a propagator definition file (.pg) was used so that every numerical integration setting was exactly the same between the programs. The 3 initial SBSS orbital locations are shown below.

	sma (km)	ecc	incl (°)	raan (°)	argp (°)	arglat (°)
Sat23851	7285.844629	0.00180596	99.417245	68.295775	52.336899	6.932798
	Position vector (km)			Velocity vector (km/s)		
	2804.845315	6658.143319	866.484967	0.78338300	-1.28412100	7.25157400
Sat31115	27911.395756	0.00052778	56.122495	56.330153	173.830096	18.204730
	10659.592444	24773.112276	7243.006494	-2.31910918	0.12604617	2.97876542
Sat25937	42166.086012	0.00026190	0.084579	95.627523	232.843615	322.886051
	22023.176541	35957.718780	-37.558623	-2.62147598	1.60653456	0.00361857

Epoch 1 Feb 2010 00:00:00.000 UTCG

4.1 Accuracy of SBSS Orbit

The first value to establish was the accuracy of the SBSS vehicle. With ground sensors, we generally know the location to sub-meter level accuracy. With space based sensors, this is generally not the case. Thus, we first establish a reference orbit from simulated observations from a notional AFSCN sensor network. The observations consist of range azimuth and elevation values mimicking data that would be present from a transponder system, or perhaps an on-board GPS receiver. Instead of generating observations anytime the sensor has access to the satellite, a more realistic situation was modeled in which bursts of observations, about 10 minutes in length (user selected) were simulated for each sensor. Collectively, the observations were then processed through the filter and smoother. The notion of a ***cycle time*** is introduced here as it was used to space the number of contacts within a longer pass duration. For instance, some of the objects had continuous 24 hour-a-day coverage of the SBSS vehicle. It's unlikely that one satellite would get a full 24 hours of dedicated coverage. Thus, the cycle time was created to fragment continuous long period coverages. Typically, a cycle time of 360 min was used. With 10 minute bursts of data, the data was randomly placed within each 360 minute window. The same process was repeated for the remaining windows.

For the SBSS in a LEO orbit, we find the following from the selected tracking. The resulting accuracy of the orbit is about 25-35 m which is reasonable for this orbital class.

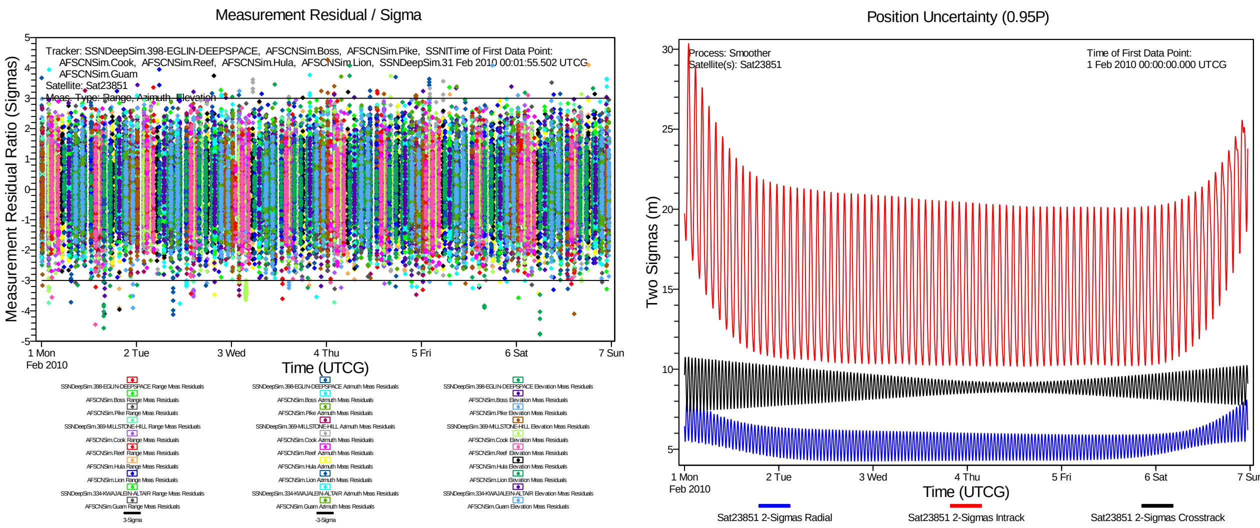

Figure 1. Residual Ratios and Position Uncertainty – LEO SBSS Orbit. The simulated data is processed to determine the general accuracy of the SBSS orbit.

For the GPS SBSS location, we ran the same analysis and find a position uncertainty of about 200 m.

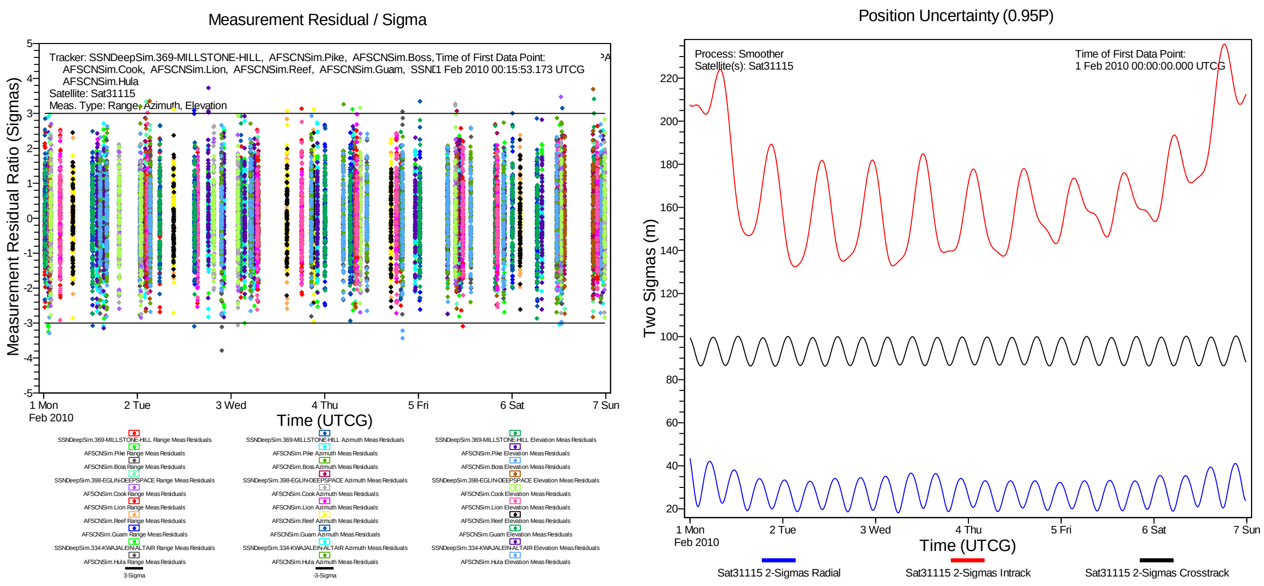

Figure 2. Residual Ratios and Position Uncertainty – GPS SBSS Orbit. The simulated data is processed to determine the general accuracy of the SBSS orbit. 10 min spacing at 360 min cycle times.

Finally, for the GEO SBSS location, we also assume a transponder-like system and arrive at the following position uncertainty. This particular test showed about a 250-300m uncertainty throughout the interval (10 min observation bursts, cycle time of 360 min).

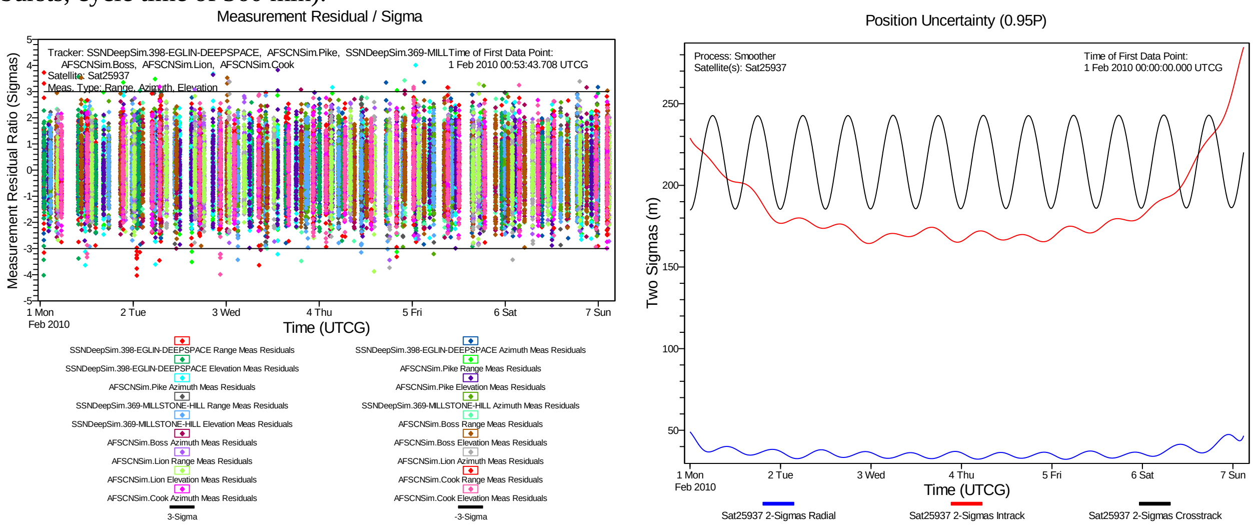

Figure 3. Residual Ratios and Position Uncertainty – GEO SBSS Orbit. The simulated data is processed to determine the general accuracy of the SBSS orbit.

We could alternatively simulate the SBSS orbits derived only from optical tracking data (SSN), although this would unnecessarily impart error into the solution. The idea is to achieve the best accuracy for the SBSS so that all measurements are more meaningful from that platform.

4.2 RSO Satellites

Five GEO Resident Space Object (RSO) satellites were simulated for the analysis and 3 were examined in detail. Orbital parameters and state vectors are listed below. The satellites were primarily selected for the GEO SBSS case to exhibit slight inclination differences, and some near singularities in observability. Obviously, the LEO and GPS tracking will introduce significantly more relative motion between the objects, but these will provide good baseline comparisons for the GEO cases. Orbital elements and position and velocity vectors are given for each satellite.

	sma (km)	ecc	incl (°)	raan (°)	argp (°)	arglat (°)
Tle-10953	42166.458908	0.00051524	14.429716	358.341814	350.562430	23.932225

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                     Position vector (km)                      Velocity vector (km/s)
           38987.662670   15436.628340   4260.614449   -1.16764004   2.75804746   0.70068008
Tle-26900  42165.971923   0.00029614    0.089886    92.834454    210.992138   100.094786
          -41101.230600   -9435.526021   65.132756   0.68868869   -2.99613792   -0.00084666
Tle-27403  42166.385605   0.00029178    0.103046    85.152027    229.317924   11.332900
           -4763.430298   41906.220865   14.905830   -3.05426201   -0.34662058   0.00542072
Epoch 1 Feb 2010 00:00:00.000 UTCG
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To determine a baseline, we examine several different properties from one satellite (tle-10953). This let us investigate the general behavior of the angles-only solution before extending the analysis to the remaining satellites.

5. IOD Observation Evaluations

The results for the IOD and the various combinations of observations were examined first, and are shown in Table 1.

Table 1. IOD Results – LEO SBSS to Satellite 10953. The various IOD answers are shown. The yellow highlighted values represent the closest solution to the known answer. Semimajor axis is the most important parameter to match. An average or median value is shown. The processing is either sequential observations, or all permutations. The spacing between the observations (30 sec is the default) and the initial range estimate are shown.

Sequence									a (km)	ecc	incl (deg)	raan (deg)	argp (deg)	arglat (deg)	# soltn	Solution Epoch		
						0	1	2	178426.0783	0.73992	5.9068	307.129	84.486	76.128	2	1 Feb	2010	0:08
						1	2	3	28691.1872	0.36317	30.3676	12.358	176.630	13.887	3	1 Feb	2010	0:09
						2	3	4	208977.0168	0.77739	5.9917	310.397	80.064	73.155	3	1 Feb	2010	0:09
						3	4	5	32373.4889	0.23688	21.5365	6.935	194.540	18.630	3	1 Feb	2010	0:10
						4	5	6	36611.7949	0.17964	16.6508	1.683	242.195	23.515	3	1 Feb	2010	0:10
						5	6	7	45552.5033	0.13759	14.6114	358.812	92.552	26.235	3	1 Feb	2010	0:11
						6	7	8	40848.7762	0.04463	14.7996	358.938	256.587	26.293	3	1 Feb	2010	0:11
						7	8	9	40567.4907	0.04838	14.9236	359.146	250.621	26.222	3	1 Feb	2010	0:12
						8	9	10	53052.1535	0.18873	12.0943	353.501	47.389	31.669	2	1 Feb	2010	0:12
						9	10	11	38484.6009	0.09055	15.6379	0.226	230.393	25.473	3	1 Feb	2010	0:13
						10	11	12	42229.7924	0.08255	13.8689	357.147	294.246	28.499	2	1 Feb	2010	0:13
						11	12	13	78375.8779	0.44747	8.7967	341.033	17.707	44.193	3	1 Feb	2010	0:14
						12	13	14	33170.1506	0.22100	18.4970	3.840	202.871	22.523	3	1 Feb	2010	0:14
	Obs		**Spacing**		**Initial Est**			**Passed**										
avg	15	seq	0.5	min	5	5	ER	13	65,950.8394	0.27368	14.8987	351.627	333.099	76.128				
med	15	seq	0.5	min	5	5	ER	13	36,611.7949	0.17964	16.6508	1.683	242.195	23.515	3	1 Feb	2010	0:10
med	15	seq	0.5	min	5	5	ER	13	36,611.7949	0.17964	16.6508	1.683	242.195	23.515	3	1 Feb	2010	0:10
med	15	seq	1	min	5	5	ER	11	42,089.7446	0.03117	14.2876	358.034	292.717	27.518	2	1 Feb	2010	0:13
med	15	seq	1.5	min	5	5	ER	9	41,917.7066	0.00435	14.6328	358.699	197.635	26.506	3	1 Feb	2010	0:11
med	15	seq	2	min	5	5	ER	7	40,890.0133	0.02575	15.0270	359.357	204.988	25.891	3	1 Feb	2010	0:11
med	15	seq	2.5	min	5	5	ER	5	40,896.9804	0.02916	15.2005	359.665	175.613	25.347	3	1 Feb	2010	0:10
med	15	seq	0.5	min	1	1	ER	13	7,288.3864	0.00319	99.3515	68.241	83.220	42.346	3	1 Feb	2010	0:10
med	15	seq	0.5	min	3	3	ER	13	36,611.7932	0.17964	16.6508	1.683	242.195	23.515	3	1 Feb	2010	0:10
med	15	seq	0.5	min	4	4	ER	13	36,611.7728	0.17964	16.6508	1.683	242.195	23.515	3	1 Feb	2010	0:10
med	15	seq	0.5	min	5	5	ER	13	36,611.7949	0.17964	16.6508	1.683	242.195	23.515	3	1 Feb	2010	0:10
med	15	seq	0.5	min	6	6	ER	13	36,611.7932	0.17964	16.6508	1.683	242.195	23.515	3	1 Feb	2010	0:10
avg	15	all	0.5	min	5	5	ER	453	43,019.6065	0.05209	14.5661	358.194	340.338	76.128				
med	15	all	0.5	min	5	5	ER	453	41,997.8230	0.00294	14.5830	358.610	227.625	26.590	3	1 Feb	2010	0:11
med	15	all	0.5	min	5	5	ER	453	41,997.8230	0.00294	14.5830	358.610	227.625	26.590	3	1 Feb	2010	0:11
med	15	all	1	min	5	5	ER	284	41,869.9970	0.00910	14.7112	358.843	150.802	26.243	3	1 Feb	2010	0:11
med	15	all	1.5	min	5	5	ER	163	41,843.6951	0.00610	14.6271	358.683	219.733	26.647	3	1 Feb	2010	0:12
med	15	all	2	min	5	5	ER	82	41,660.6949	0.01347	14.5911	358.605	247.621	26.599	3	1 Feb	2010	0:11
med	15	all	2.5	min	5	5	ER	33	41,771.8003	0.00910	14.5816	358.597	237.182	26.604	3	1 Feb	2010	0:11
med	15	all	0.5	min	1	1	ER	453	7,288.8049	0.00284	99.4002	68.230	89.780	56.337	3	1 Feb	2010	0:14
med	15	all	0.5	min	3	3	ER	453	41,997.8227	0.00294	14.5830	358.610	227.625	26.590	3	1 Feb	2010	0:11
med	15	all	0.5	min	4	4	ER	453	41,997.8227	0.00294	14.5830	358.610	227.625	26.590	3	1 Feb	2010	0:11
med	15	all	0.5	min	5	5	ER	453	41,997.8230	0.00294	14.5830	358.610	227.625	26.590	3	1 Feb	2010	0:11
med	15	all	0.5	min	6	6	ER	453	41,997.8227	0.00294	14.5830	358.610	227.625	26.590	3	1 Feb	2010	0:11
						Tle-10953		**Ans**	**42166.4589**	**0.0005**	**14.4297**	**358.342**	**350.562**	**23.932**				

Using just the first 15 observations, every 30 second spacing, 5 ER as the initial range estimate, and a sequential processing, the various combinations (1-2-3, 2-3-4, etc.) are as follows. The number of solutions is the last number in the column and the data includes the orbital parameters in the middle. Notice the variability as different observation sequences are used for the process. The answer is given with the average orbital elements. The answer (which is known ahead of time) is shown on the line labeled "Ans". The solution epoch for the various trials are given, along with the number of solutions where multiple solutions are present. The extreme variability suggests that perhaps an average of each trial is not the best approach. A sorting algorithm was implemented to find the median value, also shown below. Of course, the median becomes less meaningful as the number of selected trials/observations decreases, but for this case, it seemed to work reasonably well. All 13 trials resulted in solutions that were not immediately rejected (the code was setup to eliminate any parabolic or hyperbolic orbits, or cases where a solution was not found). Note that the closest answer is highlighted for each category. All simulated obs had 20 minute tracks from the LEO SBSS (360 min cycle time). There are times associated with some of the solutions. This is because when using a median value, the specific observations yield a solution epoch for that combination.[1] When averaging the orbital elements, no such epoch is possible. Note the variability from each trial of the angles-only solution. Some trials result in solutions that are very close to each other, some are very different, and some trials result in nonsense solutions.

Next, we selected different spacing of the observations. It appeared that the Gooding technique does not appear to suffer from closely spaced or widely spaced observations (remember this analysis uses 30 sec spacing as the minimum inter-observation interval).

Table 2. IOD Results – LEO SBSS to Satellite 26900. The various IOD answers are shown. The yellow highlighted values represent the closest solution to the known answer. Semimajor axis is the most important parameter to match. An average or median value is shown. The processing is either sequential observations, or all permutations. The spacing between the observations (30 sec is the default) and the initial range estimate are shown.

Sequence									a (km)	ecc	incl (deg)	raan (deg)	argp (deg)	arglat (deg)	# soltn	Solution Epoch		
						0	1	2	36297.0640	0.23729	1.7745	25.591	47.482	182.411	3	1 Feb	2010	0:59
						1	2	3	38212.8654	0.14713	1.0025	27.525	47.222	180.544	3	1 Feb	2010	1:00
						2	3	4	42832.5255	0.12145	1.3109	30.959	265.471	177.178	2	1 Feb	2010	1:00
						3	4	5	46288.9703	0.08891	0.7641	196.070	41.569	12.132	3	1 Feb	2010	1:01
						4	5	6	77468.1514	0.42762	4.8095	202.386	16.411	5.776	3	1 Feb	2010	1:01
						5	6	7	35504.3936	0.23380	1.4513	23.598	43.875	185.019	3	1 Feb	2010	1:02
						6	7	8	57655.3672	0.27420	1.8349	198.545	44.128	9.969	2	1 Feb	2010	1:02
						7	8	9	37476.4827	0.10585	1.4912	26.616	4.596	182.201	3	1 Feb	2010	1:03
						8	9	10	50334.2935	0.25260	0.6327	45.070	229.197	163.767	2	1 Feb	2010	1:03
						9	10	11	31982.3288	0.29355	6.1690	25.726	340.023	183.475	2	1 Feb	2010	1:04
						10	11	12	64370.8671	0.51871	5.4436	206.410	283.116	2.670	3	1 Feb	2010	1:04
						11	12	13	38454.9804	0.21361	3.5784	27.576	302.879	181.744	3	1 Feb	2010	1:05
						12	13	14	42505.3561	0.14471	1.1856	205.402	268.902	3.979	3	1 Feb	2010	1:05
	Obs		Spacing		Initial Est			Passed										
avg	15	seq	0.5	min	5	5	ER	13	46,106.4343	0.23534	2.4191	317.036	342.683	182.411				
med	15	seq	0.5	min	5	5	ER	13	38,454.9804	0.21361	3.5784	27.576	302.879	181.744	3	1 Feb	2010	1:05
med	15	seq	0.5	min	5	5	ER	13	38,454.9804	0.21361	3.5784	27.576	302.879	181.744	3	1 Feb	2010	1:05
med	15	seq	1	min	5	5	ER	11	40,238.7862	0.06897	0.0625	54.896	24.733	154.376	3	1 Feb	2010	1:05
med	15	seq	1.5	min	5	5	ER	9	40,988.5407	0.07470	1.0468	30.618	289.591	178.156	3	1 Feb	2010	1:03
med	15	seq	2	min	5	5	ER	7	41,735.5453	0.03322	0.4174	37.706	276.669	171.053	3	1 Feb	2010	1:03
med	15	seq	2.5	min	5	5	ER	5	41,991.2176	0.02436	0.2954	43.106	263.642	165.650	3	1 Feb	2010	1:03
med	15	seq	0.5	min	1	1	ER	13	7,290.0363	0.00074	99.3562	68.295	316.018	218.935	3	1 Feb	2010	1:00
med	15	seq	0.5	min	3	3	ER	13	38,454.9676	0.21361	3.5784	27.576	302.879	181.744	3	1 Feb	2010	1:05
med	15	seq	0.5	min	4	4	ER	13	38,454.6387	0.21362	3.5787	27.576	302.881	181.744	3	1 Feb	2010	1:05
med	15	seq	0.5	min	5	5	ER	13	38,454.9804	0.21361	3.5784	27.576	302.879	181.744	3	1 Feb	2010	1:05
med	15	seq	0.5	min	6	6	ER	13	38,454.9799	0.21361	3.5784	27.576	302.879	181.744	3	1 Feb	2010	1:05
avg	15	all	0.5	min	5	5	ER	453	42,870.0404	0.06603	0.7056	335.189	358.709	182.411				
med	15	all	0.5	min	5	5	ER	453	42,643.8836	0.01162	0.1179	156.468	82.687	52.028	3	1 Feb	2010	1:02
med	15	all	0.5	min	5	5	ER	453	42,643.8836	0.01162	0.1179	156.468	82.687	52.028	3	1 Feb	2010	1:02
med	15	all	1	min	5	5	ER	284	42,783.3990	0.02309	0.1041	144.252	120.608	64.244	3	1 Feb	2010	1:02
med	15	all	1.5	min	5	5	ER	163	42,879.3002	0.01859	0.1420	165.015	81.119	43.478	3	1 Feb	2010	1:02
med	15	all	2	min	5	5	ER	82	42,934.8206	0.01983	0.1327	159.980	85.887	48.512	3	1 Feb	2010	1:02
med	15	all	2.5	min	5	5	ER	33	42,826.3162	0.02272	0.0975	130.814	130.878	77.806	3	1 Feb	2010	1:02
med	15	all	0.5	min	1	1	ER	453	7,290.2214	0.00085	99.3771	68.286	331.921	229.376	3	1 Feb	2010	1:03
med	15	all	0.5	min	3	3	ER	453	42,643.8835	0.01162	0.1179	156.468	82.687	52.028	3	1 Feb	2010	1:02
med	15	all	0.5	min	4	4	ER	453	42,643.8835	0.01162	0.1179	156.468	82.687	52.028	3	1 Feb	2010	1:02
med	15	all	0.5	min	5	5	ER	453	42,643.8836	0.01162	0.1179	156.468	82.687	52.028	3	1 Feb	2010	1:02
med	15	all	0.5	min	6	6	ER	453	42,643.8835	0.01162	0.1179	156.468	82.687	52.028	3	1 Feb	2010	1:02
						Tle-26900		Ans	**42165.9719**	**0.0003**	**0.0899**	**92.834**	**210.992**	**100.095**				

Next, we can examine the effect of changing the initial range estimate. In general, the initial range estimate didn't make too much of a difference, but the smaller ranges were much worse (1 ER in particular) as it was the farthest from the actual range distance. The procedure seemed to be pretty robust to the initial estimate, not producing good answers only when then initial estimate was orders of magnitude off the actual answer. From these tests, we conclude the initial default is probably ok.

With slightly larger numbers of observations, it appears that the more widely spaced observations produce better results. The best solution from the sequential processing occurred with 15 observations, the median value, every 1 minute spacing, and initial range estimates of 5 ER. The use of semimajor axis is because it is the most influential orbital element at this stage of selecting an acceptable orbit.

[1] The mode was considered, but choosing a mode with 6 real variables proved more difficult (data bins for each), and the objective was to perform some simple operations to yield a better initial guess from the IOD solution.

The variability of the individual solutions, and success in using a median approach suggested that more trials might produce better solutions. Therefore, we tried the permutations of the observations instead of just the sequential combinations. Better results were sometimes obtained using the mode value, however because this is more lengthy to program, the median value was used once the results from the permutations were sorted. With all the permutations, it's advisable to limit the number of possibilities. For this case, if we considered all 40 observations in the first track, there would be over 9500 resulting combinations, so it was decided to leave the permutations at just 15 observations, and 453 possibilities. Too few combinations runs the risk of not finding the proper mode/median value. More combinations creates much longer run times, and is better accomplished with subsequent least-squares processing.

Unfortunately, the preceding runs didn't necessarily prove or disprove one approach over another. So two other satellites were examined to see if any trends emerged.

Table 3. IOD Results – LEO SBSS to Satellite 27403. The various IOD answers are shown. The yellow highlighted values represent the closest solution to the known answer. Semimajor axis is the most important parameter to match. An average or median value is shown. The processing is either sequential observations, or all permutations. The spacing between the observations (30 sec is the default) and the initial range estimate are shown. The "X" in solution indicates a hyperbolic orbit was found.

Sequence									a (km)	ecc	incl (deg)	raan (deg)	argp (deg)	arglat (deg)	# soltn	Solution Epoch		
						0	1	2	123375.2376	0.74572	8.0454	277.011	110.771	181.681	3	1 Feb	2010	0:07
						1	2	3	41808.4426	0.59000	23.1362	95.179	117.743	2.726	2	1 Feb	2010	0:08
						2	3	4	46226.7992	0.16644	2.5488	274.365	264.313	184.516	3	1 Feb	2010	0:08
						3	4	5	-42490.7433	1.73745	19.9754	276.927	118.141	182.819	X	0 ####	0	0:00
						4	5	6	37985.7730	0.25668	7.5113	95.462	119.217	3.222	3	1 Feb	2010	0:09
						5	6	7	42213.4745	0.28824	3.7092	96.995	105.975	2.007	3	1 Feb	2010	0:10
						6	7	8	-58774.5540	1.43175	12.3435	277.532	107.185	182.014	X	0 ####	0	0:00
						7	8	9	40798.2858	0.13650	2.4887	96.221	109.236	3.036	3	1 Feb	2010	0:11
						8	9	10	45544.4037	0.08175	2.3201	275.584	129.160	183.980	3	1 Feb	2010	0:11
						9	10	11	43439.2427	0.49672	12.4270	95.521	111.449	3.691	3	1 Feb	2010	0:12
						10	11	12	51984.2160	0.12257	6.3755	274.443	170.158	185.652	3	1 Feb	2010	0:12
						11	12	13	-279116.0413	1.09089	9.0499	278.604	103.073	181.364	X	0 ####	0	0:00
						12	13	14	-7584.4446	2.22568	41.5447	97.145	96.464	2.593	X	0 ####	0	0:00
	Obs		Spacing		Initial Est			Passed										
avg	15	seq	0.5	min	5	5	ER	9	52,597.3194	0.32051	7.6180	15.642	97.558	181.681				
med	15	seq	0.5	min	5	5	ER	9	41,808.4426	0.59000	23.1362	95.179	117.743	2.726	2	1 Feb	2010	0:08
med	15	seq	0.5	min	5	5	ER	9	41,808.4426	0.59000	23.1362	95.179	117.743	2.726	2	1 Feb	2010	0:08
med	15	seq	1	min	5	5	ER	11	42,418.4266	0.28183	3.8462	97.272	104.618	2.482	3	1 Feb	2010	0:13
med	15	seq	1.5	min	5	5	ER	9	42,473.4724	0.25023	2.9881	97.730	102.989	1.929	3	1 Feb	2010	0:12
med	15	seq	2	min	5	5	ER	7	43,107.9216	0.05812	0.5564	279.516	104.924	179.947	3	1 Feb	2010	0:11
med	15	seq	2.5	min	5	5	ER	5	44,219.3799	0.11425	1.2041	278.405	105.314	180.946	3	1 Feb	2010	0:11
med	15	seq	0.5	min	1	1	ER	13	7,383.4989	0.01571	99.5843	68.545	67.631	43.048	3	1 Feb	2010	0:10
med	15	seq	0.5	min	3	3	ER	9	41,808.4375	0.59000	23.1362	95.179	117.743	2.726	3	1 Feb	2010	0:08
med	15	seq	0.5	min	4	4	ER	9	41,808.4476	0.59000	23.1362	95.179	117.743	2.726	3	1 Feb	2010	0:08
med	15	seq	0.5	min	5	5	ER	9	41,808.4426	0.59000	23.1362	95.179	117.743	2.726	2	1 Feb	2010	0:08
med	15	seq	0.5	min	6	6	ER	9	41,805.4095	0.58995	23.1344	95.179	117.745	2.726	3	1 Feb	2010	0:08
avg	15	all	0.5	min	5	5	ER	449	46,719.0166	0.15574	2.3517	338.451	91.254	181.681				
med	15	all	0.5	min	5	5	ER	449	43,570.8008	0.07774	0.8757	278.355	106.719	181.118	3	1 Feb	2010	0:11
med	15	all	0.5	min	5	5	ER	449	43,570.8008	0.07774	0.8757	278.355	106.719	181.118	3	1 Feb	2010	0:11
med	15	all	1	min	5	5	ER	284	43,705.2153	0.05215	1.0370	276.531	118.754	183.214	3	1 Feb	2010	0:12
med	15	all	1.5	min	5	5	ER	163	43,960.4787	0.08152	1.1923	277.556	110.862	181.430	3	1 Feb	2010	0:09
med	15	all	2	min	5	5	ER	82	44,020.5260	0.09532	1.1473	277.927	107.672	181.178	3	1 Feb	2010	0:10
med	15	all	2.5	min	5	5	ER	33	43,950.9088	0.09057	1.1190	277.883	108.104	181.097	2	1 Feb	2010	0:09
med	15	all	0.5	min	1	1	ER	453	7,384.6602	0.01577	99.5708	68.554	68.636	46.550	3	1 Feb	2010	0:11
med	15	all	0.5	min	3	3	ER	449	43,570.8008	0.07774	0.8757	278.355	106.719	181.118	3	1 Feb	2010	0:11
med	15	all	0.5	min	4	4	ER	449	43,570.8008	0.07774	0.8757	278.355	106.719	181.118	3	1 Feb	2010	0:11
med	15	all	0.5	min	5	5	ER	449	43,570.8008	0.07774	0.8757	278.355	106.719	181.118	3	1 Feb	2010	0:11
med	15	all	0.5	min	6	6	ER	449	43,570.8008	0.07774	0.8757	278.355	106.719	181.118	3	1 Feb	2010	0:11
						Tle-27403		Ans	**42166.3856**	**0.0003**	**0.1030**	**85.152**	**229.318**	**11.333**				

The LEO tests showed tremendous variations with almost all test combinations. The best results seem to occur with the median value, sequential observations, slightly longer spacing between observations (1-2 min), and the default initial range estimate. The other trials sometimes performed very poorly. The initial range didn't appear to influence the results a lot. However, if the initial estimate was widely wrong, it could influence the results. These test cases are dependant on the assumptions made – 20 min bursts of data, cycled every 360 minutes.

Now with the SBSS in a GPS-like orbit, we find the following results.

Table 4. IOD Results – GPS SBSS to Satellite 10953. The various IOD answers are shown. The yellow highlighted values represent the closest solution to the known answer. Semimajor axis is the most important parameter to match. An average or median value is shown. The processing is either sequential observations, or all permutations. The spacing between the observations (30 sec is the default) and the initial range estimate are shown. The "X" in solution indicates a hyperbolic orbit was found.

Sequence									a (km)	ecc	incl (deg)	raan (deg)	argp (deg)	arglat (deg)	# soltn	Solution Epoch		
						0	1	2	27910.2905	0.00059	56.1225	56.320	172.211	163.909	2	1 Feb	2010	5:12
						1	2	3	22198.5279	0.47125	21.6819	343.606	284.743	124.459	4	1 Feb	2010	5:13
						2	3	4	-2071.8555	27.90672	21.3632	67.760	319.491	21.375	X	0 ####	0	0:00
						3	4	5	30950.2262	0.79795	33.8072	309.419	0.695	151.267	4	1 Feb	2010	5:14
						4	5	6	27915.1559	0.00067	56.1250	56.322	184.138	164.844	2	1 Feb	2010	5:14
						5	6	7	27917.3195	0.00076	56.1305	56.325	207.669	165.081	2	1 Feb	2010	5:15
						6	7	8	27911.9509	0.00061	56.1238	56.321	179.109	165.308	1	1 Feb	2010	5:15
						7	8	9	27902.5298	0.00043	56.1219	56.317	167.388	165.537	1	1 Feb	2010	5:16
						8	9	10	27910.6400	0.00058	56.1235	56.320	177.583	165.774	1	1 Feb	2010	5:16
						9	10	11	-66371.6246	1.22826	25.8113	36.741	152.822	103.099	X	0 ####	0	0:00
						10	11	12	27913.5193	0.00076	56.1185	56.320	155.600	166.237	1	1 Feb	2010	5:17
						11	12	13	27910.8655	0.00058	56.1240	56.320	180.195	166.473	1	1 Feb	2010	5:18
						12	13	14	27893.6478	0.00030	56.1201	56.313	142.351	166.699	1	1 Feb	2010	5:18
	Obs		Spacing		Initial Est			Passed										
avg	15	seq	0.5	min	5	5	ER	11	27,666.7885	0.11586	50.9635	39.991	37.426	163.909				
med	15	seq	0.5	min	5	5	ER	11	27,910.6400	0.00058	56.1235	56.320	177.583	165.774	1	1 Feb	2010	5:16
med	15	seq	0.5	min	5	5	ER	11	27,910.6400	0.00058	56.1235	56.320	177.583	165.774	1	1 Feb	2010	5:16
med	15	seq	1	min	5	5	ER	7	27,911.5986	0.00060	56.1246	56.321	183.073	165.774	1	1 Feb	2010	5:16
med	15	seq	1.5	min	5	5	ER	9	27,912.7660	0.00062	56.1250	56.321	184.955	165.542	1	1 Feb	2010	5:16
med	15	seq	2	min	5	5	ER	7	27,912.8853	0.00062	56.1250	56.321	185.074	165.775	1	1 Feb	2010	5:16
med	15	seq	2.5	min	5	5	ER	5	27,913.6146	0.00064	56.1253	56.322	186.235	165.776	1	1 Feb	2010	5:16
med	15	seq	0.5	min	1	1	ER	13	27,886.5881	0.00284	56.1720	56.326	277.639	165.100	1	1 Feb	2010	5:15
med	15	seq	0.5	min	3	3	ER	13	27,910.2966	0.00057	56.1246	56.320	183.975	165.774	1	1 Feb	2010	5:16
med	15	seq	0.5	min	4	4	ER	11	27,909.6171	0.00056	56.1241	56.320	181.401	165.308	2	1 Feb	2010	5:15
med	15	seq	0.5	min	5	5	ER	11	27,910.6400	0.00058	56.1235	56.320	177.583	165.774	1	1 Feb	2010	5:16
med	15	seq	0.5	min	6	6	ER	11	27,907.5976	0.00052	56.1229	56.319	174.429	165.539	1	1 Feb	2010	5:16
avg	15	all	0.5	min	5	5	ER	402	34,035.7117	0.26091	40.8398	30.421	240.961	163.909				
med	15	all	0.5	min	5	5	ER	402	27,913.0661	0.00063	56.1250	56.322	184.882	165.076	1	1 Feb	2010	5:15
med	15	all	0.5	min	5	5	ER	402	27,913.0661	0.00063	56.1250	56.322	184.882	165.076	1	1 Feb	2010	5:15
med	15	all	1	min	5	5	ER	261	27,913.6055	0.00064	56.1253	56.322	186.323	165.776	1	1 Feb	2010	5:16
med	15	all	1.5	min	5	5	ER	159	27,913.8523	0.00064	56.1253	56.322	186.226	165.310	1	1 Feb	2010	5:15
med	15	all	2	min	5	5	ER	78	27,914.2831	0.00065	56.1255	56.322	186.782	165.310	1	1 Feb	2010	5:15
med	15	all	2.5	min	5	5	ER	32	27,914.6918	0.00066	56.1257	56.322	187.468	165.310	1	1 Feb	2010	5:15
med	15	all	0.5	min	1	1	ER	452	27,915.0034	0.00067	56.1258	56.323	188.081	165.543	1	1 Feb	2010	5:16
med	15	all	0.5	min	3	3	ER	450	27,914.7430	0.00066	56.1257	56.322	187.505	165.543	1	1 Feb	2010	5:16
med	15	all	0.5	min	4	4	ER	400	27,913.0411	0.00063	56.1250	56.322	185.094	165.309	1	1 Feb	2010	5:15
med	15	all	0.5	min	5	5	ER	402	27,913.0661	0.00063	56.1250	56.322	184.882	165.076	1	1 Feb	2010	5:15
med	15	all	0.5	min	6	6	ER	401	27,913.0662	0.00063	56.1250	56.322	184.881	165.076	1	1 Feb	2010	5:15
						Tle-10953		Ans	**42166.4589**	**0.0005**	**14.4297**	**358.342**	**350.562**	**23.932**				

The results in Table 4-6 highlighted the degenerate solution phenomena that is still under investigation. Many of the solutions had only 1 answer while most of the other test combinations had 2-3 solutions for each case. Notice though that the results are closer to the SBSS GPS orbit than the target RSO in GEO. This suggests that angle-only techniques from space based sensors may yield solutions for space based applications that are representative of the host vehicle. In this case, the solution with the closest semimajor axis had the worst inclination.

Table 5. IOD Results – GPS SBSS to Satellite 26900. The various IOD answers are shown. The yellow highlighted values represent the closest solution to the known answer. Semimajor axis is the most important parameter to match. An average or median value is shown. The processing is either sequential observations, or all permutations. The spacing between the observations (30 sec is the default) and the initial range estimate are shown. The "X" in solution indicates a hyperbolic orbit was found.

Sequence									a (km)	ecc	incl (deg)	raan (deg)	argp (deg)	arglat (deg)	# soltn	Solution Epoch		
						0	1	2	28077.2496	0.00693	56.2334	56.040	180.325	180.305	1	1 Feb	2010	5:48
						1	2	3	37810.9439	0.26274	58.9786	46.669	181.566	180.602	2	1 Feb	2010	5:48
						2	3	4	27910.8224	0.00059	56.1246	56.320	176.798	180.770	2	1 Feb	2010	5:49
						3	4	5	27954.5075	0.00237	56.2433	56.179	182.582	181.004	2	1 Feb	2010	5:49
						4	5	6	27907.1590	0.00045	56.1209	56.327	175.326	181.235	2	1 Feb	2010	5:50
						5	6	7	28858.0000	0.03493	56.2463	55.136	180.464	181.497	3	1 Feb	2010	5:50
						6	7	8	34695.0860	0.18510	41.7521	63.654	163.792	181.806	4	1 Feb	2010	5:51
						7	8	9	-43755.6898	2.03621	12.0416	286.669	333.411	1.283	X	0 ####	0	0:00
						8	9	10	-5441.7784	6.38550	8.6160	77.711	158.737	185.445	X	0 ####	0	0:00
						9	10	11	26320.9379	0.06547	55.3375	59.072	0.526	182.301	4	1 Feb	2010	5:52
						10	11	12	20473.6534	0.59612	50.4790	81.580	351.523	181.346	5	1 Feb	2010	5:53
						11	12	13	27912.1958	0.00067	56.1462	56.303	179.238	182.867	2	1 Feb	2010	5:53
						12	13	14	20603.0165	0.54712	51.5328	79.589	352.339	181.718	4	1 Feb	2010	5:54
	Obs		**Spacing**		**Initial Est**			**Passed**										
avg	15	seq	0.5	min	5	5	ER	11	28,047.5975	0.15477	54.1086	60.624	356.771	180.305				
med	15	seq	0.5	min	5	5	ER	11	27,910.8224	0.00059	56.1246	56.320	176.798	180.770	2	1 Feb	2010	5:49
med	15	seq	0.5	min	5	5	ER	11	27,910.8224	0.00059	56.1246	56.320	176.798	180.770	2	1 Feb	2010	5:49
med	15	seq	1	min	5	5	ER	9	26,601.0162	0.52028	12.9275	95.872	345.029	180.632	3	1 Feb	2010	5:50
med	15	seq	1.5	min	5	5	ER	6	42,031.4728	0.09900	3.7292	281.840	117.869	0.654	3	1 Feb	2010	5:50
med	15	seq	2	min	5	5	ER	6	31,353.3924	0.23415	10.7334	95.332	356.068	180.817	4	1 Feb	2010	5:50
med	15	seq	2.5	min	5	5	ER	5	35,410.2852	0.14324	5.0512	97.826	353.235	180.970	3	1 Feb	2010	5:51
med	15	seq	0.5	min	1	1	ER	13	27,679.3697	0.00892	55.6050	57.001	4.329	180.302	2	1 Feb	2010	5:48
med	15	seq	0.5	min	3	3	ER	13	27,643.9602	0.01103	55.1161	57.412	7.345	180.994	2	1 Feb	2010	5:49
med	15	seq	0.5	min	4	4	ER	12	27,800.5031	0.00363	56.0849	56.482	0.565	180.535	2	1 Feb	2010	5:48
med	15	seq	0.5	min	5	5	ER	11	27,910.8224	0.00059	56.1246	56.320	176.798	180.770	2	1 Feb	2010	5:49
med	15	seq	0.5	min	6	6	ER	10	27,894.8195	0.00025	55.9814	56.439	47.190	182.865	2	1 Feb	2010	5:53
avg	15	all	0.5	min	5	5	ER	379	66,083.2423	0.23935	15.7171	14.527	29.177	180.305				
med	15	all	0.5	min	5	5	ER	379	35,477.3562	0.11567	7.6093	96.509	12.068	180.603	3	1 Feb	2010	5:49
med	15	all	0.5	min	5	5	ER	379	35,477.3562	0.11567	7.6093	96.509	12.068	180.603	3	1 Feb	2010	5:49
med	15	all	1	min	5	5	ER	252	37,760.3762	0.09527	2.5758	98.852	348.955	181.036	3	1 Feb	2010	5:51
med	15	all	1.5	min	5	5	ER	151	39,332.5544	0.05218	2.1740	98.961	359.095	180.830	3	1 Feb	2010	5:50
med	15	all	2	min	5	5	ER	82	40,817.7541	0.03328	0.4012	99.107	337.264	181.745	3	1 Feb	2010	5:51
med	15	all	2.5	min	5	5	ER	33	43,729.4087	0.02558	1.2033	280.948	14.807	0.607	3	1 Feb	2010	5:51
med	15	all	0.5	min	1	1	ER	449	24,235.3368	0.19378	49.6275	67.469	1.070	182.240	7	1 Feb	2010	5:53
med	15	all	0.5	min	3	3	ER	445	24,234.9667	0.23234	44.0642	72.757	2.590	180.797	3	1 Feb	2010	5:49
med	15	all	0.5	min	4	4	ER	414	34,384.4574	0.16857	6.0627	97.415	353.093	180.953	3	1 Feb	2010	5:51
med	15	all	0.5	min	5	5	ER	379	35,477.3562	0.11567	7.6093	96.509	12.068	180.603	3	1 Feb	2010	5:49
med	15	all	0.5	min	6	6	ER	374	35,713.4628	0.11898	6.2883	97.179	4.217	181.111	4	1 Feb	2010	5:51
						Tle-26900		**Ans**	**42165.9719**	**0.0003**	**0.0899**	**92.834**	**210.992**	**100.095**				

Table 6. IOD Results – GPS SBSS to Satellite 27403. The various IOD answers are shown. The yellow highlighted values represent the closest solution to the known answer. Semimajor axis is the most important parameter to match. An average or median value is shown. The processing is either sequential observations, or all permutations. The spacing between the observations (30 sec is the default) and the initial range estimate are shown. The "X" in solution indicates a hyperbolic orbit was found.

Sequence									a (km)	ecc	incl (deg)	raan (deg)	argp (deg)	arglat (deg)	# soltn	Solution Epoch		
						0	1	2	27911.3575	0.00052	56.1256	56.328	172.811	23.360	2	1 Feb	2010	0:11
						1	2	3	54134.1745	0.28108	2.0366	301.076	116.929	157.236	4	1 Feb	2010	0:11
						2	3	4	27762.0291	0.00962	55.4618	56.798	182.471	23.623	3	1 Feb	2010	0:12
						3	4	5	27867.4025	0.00310	56.0724	56.484	174.741	23.984	1	1 Feb	2010	0:12
						4	5	6	348997.0659	0.85682	22.0226	274.253	171.705	193.051	5	1 Feb	2010	0:13
						5	6	7	28697.0768	0.19685	27.7919	74.664	178.433	16.702	3	1 Feb	2010	0:13
						6	7	8	26852.3659	0.27322	37.5277	75.616	168.897	15.165	4	1 Feb	2010	0:14
						7	8	9	-32089.3870	2.56250	30.7465	277.557	160.548	193.460	X	0 ####	0	0:00
						8	9	10	43426.4897	0.02821	0.7107	272.895	157.885	187.526	6	1 Feb	2010	0:15
						9	10	11	29092.2599	0.05931	59.5812	53.394	2.423	26.811	3	1 Feb	2010	0:15
						10	11	12	-72876.6860	1.70938	28.9539	276.855	166.033	193.894	X	0 ####	0	0:00
						11	12	13	27939.9196	0.00179	56.7035	56.387	56.982	25.873	2	1 Feb	2010	0:16
						12	13	14	27910.8304	0.00055	56.1258	56.330	172.607	26.149	2	1 Feb	2010	0:17
	Obs		**Spacing**		**Initial Est**			**Passed**										
avg	15	seq	0.5	min	5	5	ER	11	60,962.8156	0.15555	39.1054	23.111	108.717	23.360				
med	15	seq	0.5	min	5	5	ER	11	27,911.3575	0.00052	56.1256	56.328	172.811	23.360	2	1 Feb	2010	0:11
med	15	seq	0.5	min	5	5	ER	11	27,911.3575	0.00052	56.1256	56.328	172.811	23.360	2	1 Feb	2010	0:11
med	15	seq	1	min	5	5	ER	7	28,799.7629	0.23509	26.4789	77.643	173.322	14.917	3	1 Feb	2010	0:13
med	15	seq	1.5	min	5	5	ER	8	33,839.2168	0.13369	12.2913	80.989	179.265	15.441	4	1 Feb	2010	0:15
med	15	seq	2	min	5	5	ER	7	31,140.3131	0.18006	18.7951	78.881	177.528	15.860	3	1 Feb	2010	0:15
med	15	seq	2.5	min	5	5	ER	5	36,717.2000	0.08527	7.1086	82.782	180.978	15.091	3	1 Feb	2010	0:14
med	15	seq	0.5	min	1	1	ER	13	27,900.9844	0.00113	56.0770	56.358	178.943	24.975	4	1 Feb	2010	0:14
med	15	seq	0.5	min	3	3	ER	13	27,909.6912	0.00062	56.1235	56.334	173.101	26.147	2	1 Feb	2010	0:17
med	15	seq	0.5	min	4	4	ER	9	27,912.9647	0.00043	56.1387	56.325	168.100	25.919	3	1 Feb	2010	0:16
med	15	seq	0.5	min	5	5	ER	11	27,911.3575	0.00052	56.1256	56.328	172.811	23.360	2	1 Feb	2010	0:11
med	15	seq	0.5	min	6	6	ER	11	27,927.1616	0.00057	56.2809	56.295	37.535	23.835	2	1 Feb	2010	0:12
avg	15	all	0.5	min	5	5	ER	423	48,117.2576	0.15585	14.0913	22.569	75.691	23.360				
med	15	all	0.5	min	5	5	ER	423	37,778.3990	0.07325	5.3171	84.163	177.436	14.222	3	1 Feb	2010	0:14
med	15	all	0.5	min	5	5	ER	423	37,778.3990	0.07325	5.3171	84.163	177.436	14.222	3	1 Feb	2010	0:14
med	15	all	1	min	5	5	ER	272	38,269.4017	0.05893	4.8603	83.373	182.640	15.192	3	1 Feb	2010	0:14
med	15	all	1.5	min	5	5	ER	161	39,375.7385	0.04312	3.3231	84.252	181.449	14.580	3	1 Feb	2010	0:13
med	15	all	2	min	5	5	ER	82	39,291.5670	0.05073	3.1930	85.560	173.884	13.525	3	1 Feb	2010	0:14
med	15	all	2.5	min	5	5	ER	33	38,888.1657	0.04466	4.2104	82.707	189.373	15.735	3	1 Feb	2010	0:13
med	15	all	0.5	min	1	1	ER	453	27,485.2472	0.02789	54.2982	57.812	181.304	24.307	3	1 Feb	2010	0:14
med	15	all	0.5	min	3	3	ER	449	36,717.5489	0.08526	7.1081	82.782	180.978	15.091	3	1 Feb	2010	0:14
med	15	all	0.5	min	4	4	ER	427	37,653.2335	0.30151	67.6285	43.156	4.010	30.384	5	1 Feb	2010	0:14
med	15	all	0.5	min	5	5	ER	423	37,778.3990	0.07325	5.3171	84.163	177.436	14.222	3	1 Feb	2010	0:14
med	15	all	0.5	min	6	6	ER	423	37,722.2596	0.06932	5.5810	83.379	180.894	15.111	3	1 Feb	2010	0:15
					Tle-27403			**Ans**	**42166.3856**	**0.0003**	**0.1030**	**85.152**	**229.318**	**11.333**				

The GPS tests also showed tremendous variations with almost all the parameters. The best results seem to occur with the median value, sequential observations, slightly longer spacing between observations (2 min), and the default ER initial range estimate. The other trials sometimes performed very poorly. These cases also used 20 min bursts of data, cycled every 360 minutes.

And finally, we place the SBSS in GEO orbit. The IOD results are as follows:

Table 7. IOD Results – GEO SBSS to Satellite 10953. The various IOD answers are shown. The yellow highlighted values represent the closest solution to the known answer. Semimajor axis is the most important parameter to match. An average or median value is shown. The processing is either sequential observations, or all permutations. The spacing between the observations (30 sec is the default) and the initial range estimate are shown.

Sequence									a (km)	ecc	incl (deg)	raan (deg)	argp (deg)	arglat (deg)	# soltn	Solution Epoch		
						0	1	2	42162.6288	0.00026	0.0852	95.426	215.204	345.849	1	1 Feb	2010	1:30
						1	2	3	42647.4381	0.01501	0.1425	359.672	65.391	81.201	2	1 Feb	2010	1:31
						2	3	4	42050.9396	0.00126	0.1446	52.457	247.788	28.775	1	1 Feb	2010	1:31
						3	4	5	42105.9185	0.00249	0.1903	34.248	0.665	46.885	4	1 Feb	2010	1:32
						4	5	6	41994.7184	0.00188	0.1786	45.937	244.326	35.438	4	1 Feb	2010	1:32
						5	6	7	42150.0134	0.00053	0.0879	96.582	177.784	345.326	1	1 Feb	2010	1:33
						6	7	8	42162.4040	0.00027	0.0853	95.432	214.086	346.594	1	1 Feb	2010	1:33
						7	8	9	42163.5193	0.00025	0.0851	95.680	217.355	346.472	3	1 Feb	2010	1:34
						8	9	10	40647.6762	0.03526	0.4642	69.014	184.676	12.868	1	1 Feb	2010	1:34
						9	10	11	42175.8980	0.00032	0.0825	94.915	289.084	347.483	3	1 Feb	2010	1:35
						10	11	12	42163.6054	0.00025	0.0851	95.685	217.728	346.844	1	1 Feb	2010	1:35
						11	12	13	42162.7589	0.00026	0.0852	95.472	215.405	347.181	1	1 Feb	2010	1:36
						12	13	14	64986.4463	0.37678	2.8591	310.448	117.005	124.720	2	1 Feb	2010	1:36
Obs		**Spacing**			**Initial Est**			**Passed**										
avg	15	seq	0.5	min	5	5	ER	13	43,813.3819	0.03345	0.3520	63.151	295.884	345.849				
med	15	seq	0.5	min	5	5	ER	13	42,162.4040	0.00027	0.0853	95.432	214.086	346.594	1	1 Feb	2010	1:33
med	15	seq	0.5	min	5	5	ER	13	42,162.4040	0.00027	0.0853	95.432	214.086	346.594	1	1 Feb	2010	1:33
med	15	seq	1	min	5	5	ER	11	42,107.7569	0.00131	0.0971	88.731	176.538	353.895	1	1 Feb	2010	1:36
med	15	seq	1.5	min	5	5	ER	9	42,163.0257	0.00026	0.0852	95.955	212.628	345.948	1	1 Feb	2010	1:33
med	15	seq	2	min	5	5	ER	7	41,607.8378	0.00448	0.6180	22.593	288.180	57.685	3	1 Feb	2010	1:33
med	15	seq	2.5	min	5	5	ER	5	42,166.8386	0.00023	0.0844	95.814	235.080	346.088	2	1 Feb	2010	1:33
med	15	seq	0.5	min	1	1	ER	13	41,214.5921	0.01150	0.7316	26.833	245.874	53.240	8	1 Feb	2010	1:33
med	15	seq	0.5	min	3	3	ER	13	42,160.8235	0.00029	0.0856	95.090	209.936	346.935	1	1 Feb	2010	1:33
med	15	seq	0.5	min	4	4	ER	11	42,162.8500	0.00026	0.0852	95.455	215.981	347.198	1	1 Feb	2010	1:36
med	15	seq	0.5	min	5	5	ER	13	42,162.4040	0.00027	0.0853	95.432	214.086	346.594	1	1 Feb	2010	1:33
med	15	seq	0.5	min	6	6	ER	13	42,163.1919	0.00026	0.0851	95.628	216.181	346.524	1	1 Feb	2010	1:34
avg	15	all	0.5	min	5	5	ER	439	42,876.9565	0.03329	3.5332	58.486	270.511	345.849				
med	15	all	0.5	min	5	5	ER	439	42,163.5193	0.00025	0.0851	95.680	217.355	346.472	3	1 Feb	2010	1:34
med	15	all	0.5	min	5	5	ER	439	42,163.5193	0.00025	0.0851	95.680	217.355	346.472	3	1 Feb	2010	1:34
med	15	all	1	min	5	5	ER	280	42,162.9488	0.00027	0.0852	91.496	250.030	350.637	1	1 Feb	2010	1:34
med	15	all	1.5	min	5	5	ER	163	42,163.3598	0.00026	0.0851	96.137	212.671	345.767	1	1 Feb	2010	1:33
med	15	all	2	min	5	5	ER	82	42,164.6929	0.00025	0.0849	96.631	214.983	345.150	4	1 Feb	2010	1:32
med	15	all	2.5	min	5	5	ER	33	42,165.2488	0.00024	0.0848	97.277	212.058	344.632	2	1 Feb	2010	1:33
med	15	all	0.5	min	1	1	ER	452	42,163.9047	0.00026	0.0850	96.568	211.610	345.338	2	1 Feb	2010	1:33
med	15	all	0.5	min	3	3	ER	451	42,162.7176	0.00027	0.0852	95.926	211.443	345.977	1	1 Feb	2010	1:33
med	15	all	0.5	min	4	4	ER	442	42,163.3053	0.00026	0.0851	96.334	210.757	345.570	2	1 Feb	2010	1:33
med	15	all	0.5	min	5	5	ER	439	42,163.5193	0.00025	0.0851	95.680	217.355	346.472	3	1 Feb	2010	1:34
med	15	all	0.5	min	6	6	ER	439	42,163.6793	0.00026	0.0850	96.171	213.914	345.608	2	1 Feb	2010	1:32
					Tle-10953			**Ans**	**42166.4589**	**0.0005**	**14.4297**	**358.342**	**350.562**	**23.932**				

Table 8. IOD Results – GEO SBSS to Satellite 26900. The various IOD answers are shown. The yellow highlighted values represent the closest solution to the known answer. Semimajor axis is the most important parameter to match. An average or median value is shown. The processing is either sequential observations, or all permutations. The spacing between the observations (30 sec is the default) and the initial range estimate are shown.

Sequence									a (km)	ecc	incl (deg)	raan (deg)	argp (deg)	arglat (deg)	# soltn	Solution Epoch		
						0	1	2	42170.7172	0.00012	0.0852	95.787	230.321	50.308	1	1 Feb	2010	5:49
						1	2	3	42165.5786	0.00021	0.0852	95.789	225.632	50.431	1	1 Feb	2010	5:49
						2	3	4	42165.9919	0.00022	0.0852	95.789	246.800	50.557	1	1 Feb	2010	5:50
						3	4	5	42135.8217	0.00135	0.0852	95.791	276.466	50.679	1	1 Feb	2010	5:50
						4	5	6	42164.0889	0.00023	0.0852	95.792	219.633	50.804	1	1 Feb	2010	5:51
						5	6	7	42173.5058	0.00007	0.0852	95.789	241.036	50.931	2	1 Feb	2010	5:51
						6	7	8	41836.7110	0.01267	0.0852	95.809	280.666	51.036	2	1 Feb	2010	5:52
						7	8	9	42166.6781	0.00019	0.0852	95.793	231.541	51.179	6	1 Feb	2010	5:52
						8	9	10	42164.8326	0.00022	0.0852	95.792	229.368	51.305	1	1 Feb	2010	5:53
						9	10	11	41664.3968	0.02002	0.0853	95.832	282.091	51.386	1	1 Feb	2010	5:53
						10	11	12	42169.2881	0.00015	0.0852	95.795	236.423	51.553	1	1 Feb	2010	5:54
						11	12	13	42166.1711	0.00020	0.0852	95.798	236.106	51.675	2	1 Feb	2010	5:54
						12	13	14	42168.1615	0.00017	0.0852	95.796	234.848	51.801	1	1 Feb	2010	5:55
	Obs		Spacing		Initial Est			Passed										
avg	15	seq	0.5	min	5	5	ER	13	42,100.9187	0.00276	0.0852	95.796	243.918	50.308				
med	15	seq	0.5	min	5	5	ER	13	42,165.5786	0.00021	0.0852	95.789	225.632	50.431	1	1 Feb	2010	5:49
med	15	seq	0.5	min	5	5	ER	13	42,165.5786	0.00021	0.0852	95.789	225.632	50.431	1	1 Feb	2010	5:49
med	15	seq	1	min	5	5	ER	10	42,166.1218	0.00020	0.0852	95.789	227.426	50.556	1	1 Feb	2010	5:50
med	15	seq	1.5	min	5	5	ER	9	42,171.9533	0.00010	0.0852	95.793	245.652	51.428	1	1 Feb	2010	5:53
med	15	seq	2	min	5	5	ER	7	42,157.0164	0.00036	0.0852	95.792	226.752	51.056	1	1 Feb	2010	5:52
med	15	seq	2.5	min	5	5	ER	5	42,067.8333	0.00191	0.0852	95.807	219.974	51.055	1	1 Feb	2010	5:52
med	15	seq	0.5	min	1	1	ER	13	42,168.1427	0.00017	0.0852	95.796	235.023	51.801	1	1 Feb	2010	5:55
med	15	seq	0.5	min	3	3	ER	13	42,167.2961	0.00018	0.0852	95.792	225.924	51.179	1	1 Feb	2010	5:52
med	15	seq	0.5	min	4	4	ER	13	42,166.6800	0.00019	0.0852	95.793	231.525	51.179	5	1 Feb	2010	5:52
med	15	seq	0.5	min	5	5	ER	13	42,165.5786	0.00021	0.0852	95.789	225.632	50.431	1	1 Feb	2010	5:49
med	15	seq	0.5	min	6	6	ER	13	42,166.6800	0.00019	0.0852	95.793	231.525	51.179	1	1 Feb	2010	5:52
avg	15	all	0.5	min	5	5	ER	446	42,131.7617	0.00185	0.0853	95.827	275.493	50.308				
med	15	all	0.5	min	5	5	ER	446	42,167.3051	0.00018	0.0852	95.796	234.826	51.802	1	1 Feb	2010	5:55
med	15	all	0.5	min	5	5	ER	446	42,167.3051	0.00018	0.0852	95.796	234.826	51.802	1	1 Feb	2010	5:55
med	15	all	1	min	5	5	ER	277	42,171.9400	0.00010	0.0852	95.792	244.172	51.554	1	1 Feb	2010	5:54
med	15	all	1.5	min	5	5	ER	158	42,178.8566	0.00005	0.0852	95.787	342.038	50.681	1	1 Feb	2010	5:50
med	15	all	2	min	5	5	ER	79	42,186.9675	0.00017	0.0852	95.788	22.492	51.306	1	1 Feb	2010	5:53
med	15	all	2.5	min	5	5	ER	31	42,199.2826	0.00038	0.0852	95.785	32.061	50.806	1	1 Feb	2010	5:51
med	15	all	0.5	min	1	1	ER	453	42,166.5770	0.00019	0.0852	95.796	234.083	51.803	1	1 Feb	2010	5:55
med	15	all	0.5	min	3	3	ER	452	42,167.3051	0.00018	0.0852	95.796	234.826	51.802	1	1 Feb	2010	5:55
med	15	all	0.5	min	4	4	ER	448	42,167.4686	0.00018	0.0852	95.796	235.456	51.676	1	1 Feb	2010	5:54
med	15	all	0.5	min	5	5	ER	446	42,167.3051	0.00018	0.0852	95.796	234.826	51.802	1	1 Feb	2010	5:55
med	15	all	0.5	min	6	6	ER	447	42,166.6800	0.00019	0.0852	95.793	231.525	51.179	1	1 Feb	2010	5:52
						Tle-26900		**Ans**	**42165.9719**	**0.0003**	**0.0899**	**92.834**	**210.992**	**100.095**				

Table 9. IOD Results – GEO SBSS to Satellite 27403. The various IOD answers are shown. The yellow highlighted values represent the closest solution to the known answer. Semimajor axis is the most important parameter to match. An average or median value is shown. The processing is either sequential observations, or all permutations. The spacing between the observations (30 sec is the default) and the initial range estimate are shown. The "X" in solution indicates a hyperbolic orbit was found.

Sequence									a (km)	ecc	incl (deg)	raan (deg)	argp (deg)	arglat (deg)	# soltn	Solution Epoch		
						0	1	2	42172.4493	0.00013	0.0848	95.629	275.015	2.473	1	1 Feb	2010	2:37
						1	2	3	42165.5591	0.00023	0.0848	95.629	227.581	2.598	1	1 Feb	2010	2:38
						2	3	4	42164.9067	0.00024	0.0848	95.630	225.342	2.723	1	1 Feb	2010	2:38
						3	4	5	42170.3153	0.00014	0.0848	95.629	254.818	2.849	4	1 Feb	2010	2:39
						4	5	6	-157883.1237	1.28041	0.1046	97.905	5.982	352.035	X	0 ####	0	0:00
						5	6	7	42164.7775	0.00025	0.0848	95.630	224.860	3.098	4	1 Feb	2010	2:40
						6	7	8	42166.1279	0.00020	0.0848	95.629	230.752	3.228	2	1 Feb	2010	2:40
						7	8	9	42165.1148	0.00024	0.0848	95.630	225.845	3.349	4	1 Feb	2010	2:41
						8	9	10	42203.0695	0.00073	0.0848	95.630	3.308	3.475	1	1 Feb	2010	2:41
						9	10	11	42167.4120	0.00020	0.0848	95.630	234.397	3.599	2	1 Feb	2010	2:42
						10	11	12	42168.0921	0.00018	0.0848	95.630	238.701	3.726	1	1 Feb	2010	2:42
						11	12	13	42164.4654	0.00026	0.0848	95.631	223.916	3.849	2	1 Feb	2010	2:43
						12	13	14	42165.5832	0.00023	0.0848	95.630	227.490	3.975	1	1 Feb	2010	2:43
	Obs		Spacing		Initial Est			Passed										
avg	15	seq	0.5	min	5	5	ER	12	42,169.8227	0.00025	0.0848	95.630	246.002	2.473				
med	15	seq	0.5	min	5	5	ER	12	42,165.5832	0.00023	0.0848	95.630	227.490	3.975	1	1 Feb	2010	2:43
med	15	seq	0.5	min	5	5	ER	12	42,165.5832	0.00023	0.0848	95.630	227.490	3.975	1	1 Feb	2010	2:43
med	15	seq	1	min	5	5	ER	11	42,164.5026	0.00026	0.0848	95.631	223.845	3.222	2	1 Feb	2010	2:40
med	15	seq	1.5	min	5	5	ER	9	42,164.2745	0.00024	0.0848	95.629	223.767	3.727	1	1 Feb	2010	2:42
med	15	seq	2	min	5	5	ER	7	42,163.9081	0.00024	0.0848	95.629	222.909	3.479	1	1 Feb	2010	2:41
med	15	seq	2.5	min	5	5	ER	5	42,163.0214	0.00025	0.0848	95.628	220.748	3.482	1	1 Feb	2010	2:41
med	15	seq	0.5	min	1	1	ER	13	44,238.9558	0.04728	0.0857	95.673	13.351	3.674	5	1 Feb	2010	2:42
med	15	seq	0.5	min	3	3	ER	12	42,165.1147	0.00024	0.0848	95.630	225.845	3.349	1	1 Feb	2010	2:41
med	15	seq	0.5	min	4	4	ER	13	42,164.9067	0.00024	0.0848	95.630	225.342	2.723	1	1 Feb	2010	2:38
med	15	seq	0.5	min	5	5	ER	12	42,165.5832	0.00023	0.0848	95.630	227.490	3.975	1	1 Feb	2010	2:43
med	15	seq	0.5	min	6	6	ER	13	42,164.7669	0.00024	0.0848	95.630	225.106	3.476	1	1 Feb	2010	2:41
avg	15	all	0.5	min	5	5	ER	451	45,689.9450	0.03088	0.0852	95.630	241.193	2.473				
med	15	all	0.5	min	5	5	ER	451	42,164.9311	0.00025	0.0848	95.630	225.151	3.348	2	1 Feb	2010	2:41
med	15	all	0.5	min	5	5	ER	451	42,164.9311	0.00025	0.0848	95.630	225.151	3.348	2	1 Feb	2010	2:41
med	15	all	1	min	5	5	ER	284	42,164.9350	0.00023	0.0848	95.629	225.776	2.600	1	1 Feb	2010	2:38
med	15	all	1.5	min	5	5	ER	163	42,165.1307	0.00025	0.0848	95.631	225.524	3.221	1	1 Feb	2010	2:40
med	15	all	2	min	5	5	ER	82	42,165.2593	0.00025	0.0848	95.631	225.767	3.095	1	1 Feb	2010	2:40
med	15	all	2.5	min	5	5	ER	33	42,165.5727	0.00025	0.0848	95.631	226.662	3.095	3	1 Feb	2010	2:40
med	15	all	0.5	min	1	1	ER	453	42,164.9819	0.00026	0.0848	95.631	224.884	3.345	1	1 Feb	2010	2:41
med	15	all	0.5	min	3	3	ER	452	42,164.9199	0.00023	0.0848	95.629	225.768	2.600	1	1 Feb	2010	2:38
med	15	all	0.5	min	4	4	ER	448	42,164.9311	0.00025	0.0848	95.630	225.151	3.348	3	1 Feb	2010	2:41
med	15	all	0.5	min	5	5	ER	451	42,164.9311	0.00025	0.0848	95.630	225.151	3.348	2	1 Feb	2010	2:41
med	15	all	0.5	min	6	6	ER	446	42,164.9199	0.00023	0.0848	95.629	225.768	2.600	1	1 Feb	2010	2:38
					Tle-27403			Ans	42166.3856	0.0003	0.1030	85.152	229.318	11.333				

Unlike the previous SBSS locations, the GEO tests appeared to be relatively consistent with almost all trials – possibly an artifact of the solution reverting to the SBSS location (the degenerate solution). The best results seem to occur with the median value, all permutations, slightly longer spacing between observations (2 min), and the default ER initial range estimate. The sequential trials performed reasonably, but were not as consistent as the permutations. These results used 20 min bursts of data, cycled every 360 minutes.

One goal of the GEO tests was to determine if the results were any better when the RSO satellite had some relative motion with the SBSS so the orbital elements could be observed. 10953 was the only satellite to have a modest inclination and therefore an expected relative motion difference. There was a little improvement in the results of 10953, but additional study is warranted as this was the only satellite to have some degree of relative motion with the SBSS.

In general, the results show that the IOD is strongly influenced by the orbit of the SBSS satellite. While the permutations approach seemed to do best, the LEO performed better with the sequential observations. A little additional spacing between the observations (initially at 30 sec spacing) seemed to perform better, although not always. The initial estimate did not appear to be a major factor in the solutions. Initial estimates that seemed rather far away from the answer still converged. The median value worked well in almost all cases and is recommended for use in applications and future analysis. Of course, *none* of the results obtained the exact initial orbit, but the process is intended only to get an approximate answer with which to start the orbit determination process.

6. OD Results – Space Based Observations Only

Whatever the form of the IOD processing, the main objective is that the result enables the OD to successfully recover the proper orbit from the IOD initial guess. This section examines the expected accuracy from an OD performed on the simulated data, using the exact initial orbit (no IOD). While it's unrealistic to know the initial orbit so well, it simply isolates an aspect of the problem. To save space, only a representative result is shown from each run, and the remaining cases are summarized in a table. The residual ratios appear as follows.

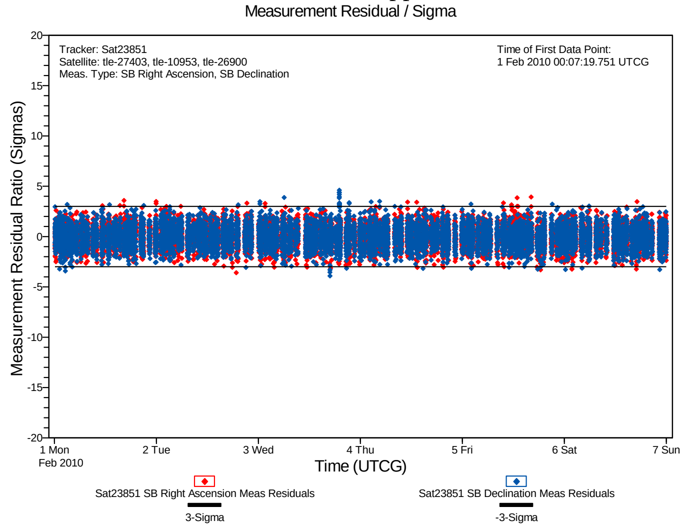

Figure 4. Filter Residual Ratios LEO SBSS tracker and GEO RSO satellites. This plot shows the residual ratios for the SBSS scenario. This case has the exact initial satellite state vectors from simulated data with all errors turned on. There are 3 satellites in the filter model here.

The general position uncertainty showed basically average performance, albeit slightly jagged response through the remainder of the interval.

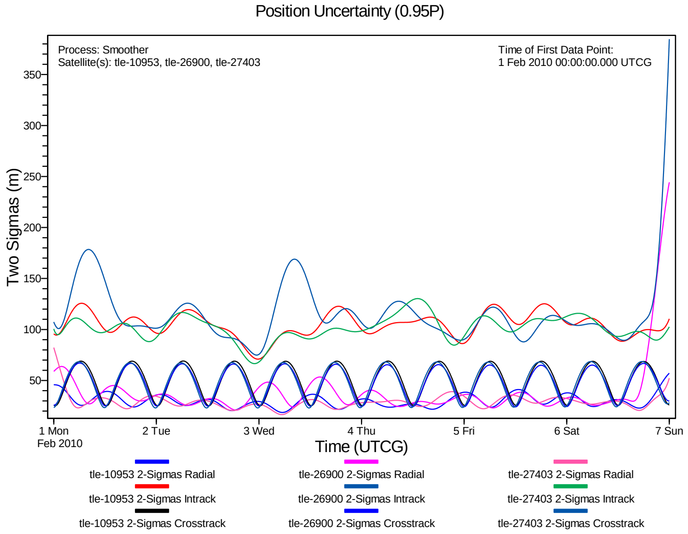

Figure 5. Smoother Position Uncertainty LEO SBSS tracker and GEO RSO satellite. This plot shows the smoother position uncertainty for the SBSS scenario. This case has the exact initial satellite state vectors from simulated data with all errors turned on. There are 3 satellites in the filter model here.

To save space, the various trials were averaged to determine the approximate uncertainty level. The average accuracy is determined by removing the 1st and last days from the results because they sometimes had additional uncertainty. Although the 1st day appeared well behaved in almost all cases, it's not likely this would always be the case as there is some additional uncertainty at the beginning.

Table 10. Average Smoother Position Uncertainty with SBSS tracker. This table shows the position uncertainties for the SBSS scenario. This case has the exact initial satellite state vectors from simulated data with all errors. The last trial is essentially for continuous observations.

SBSS	Satellite	SBSS obs time 5	20	60	360	Avg Acc (m)	Cycle Time (min)
LEO	10953	x				422	360
LEO	26900	x				508	360
LEO	27403	x				438	360
LEO	10953		x			137	180
LEO	26900		x			138	180
LEO	27403		x			130	180
LEO	10953		x			122	360
LEO	26900		x			126	360
LEO	27403		x			122	360
LEO	10953		x			136	720
LEO	26900		x			159	720
LEO	27403		x			132	720
LEO	10953		x			133	1440
LEO	26900		x			132	1440
LEO	27403		x			130	1440
LEO	10953			x		40	360
LEO	26900			x		41	360
LEO	27403			x		41	360
LEO	10953				x	26	360
LEO	26900				x	27	360
LEO	27403				x	27	360

SBSS	Satellite	SBSS obs time 5	20	60	360	Avg Acc (m)	Cycle Time (min)
GPS	10953	x				1374	360
GPS	26900	x				1386	360
GPS	27403	x				1665	360
GPS	10953		x			383	180
GPS	26900		x			357	180
GPS	27403		x			383	180
GPS	10953		x			763	360
GPS	26900		x			797	360
GPS	27403		x			781	360
GPS	10953		x			2259	720
GPS	26900		x			2558	720
GPS	27403		x			2401	720
GPS	10953		x			2155	1440
GPS	26900		x			1502	1440
GPS	27403		x			2445	1440
GPS	10953			x		717	360
GPS	26900			x		374	360
GPS	27403			x		368	360
GPS	10953				x	30	360
GPS	26900				x	38	360
GPS	27403				x	34	360

SBSS	Satellite	SBSS obs time 5	20	60	360	Avg Acc (m)	Cycle Time (min)
GEO	10953	x				1468	360
GEO	26900	x				1975	360
GEO	27403	x				1764	360
GEO	10953		x			159	180
GEO	26900		x			232	180
GEO	27403		x			178	180
GEO	10953		x			863	360
GEO	26900		x			1616	360
GEO	27403		x			1083	360
GEO	10953		x			1813	720
GEO	26900		x			2413	720
GEO	27403		x			2214	720
GEO	10953		x			2451	1440
GEO	26900		x			2458	1440
GEO	27403		x			6122	1440
GEO	10953			x		177	360
GEO	26900			x		294	360
GEO	27403			x		207	360
GEO	10953				x	96	360
GEO	26900				x	101	360
GEO	27403				x	96	360

The results were as expected. More observations (360 minutes, 360 min cycle time, or continuous observations) proved better in every case. Also, as the cycle time increased, the accuracy decreased with constant pass lengths. It was somewhat surprising that the better results were often from the LEO orbit, although this paper did not look extensively at sensor design, visible magnitude, detection, pointing requirements, etc. However, obtaining long duration observations from a LEO is likely more difficult than from a GPS or GEO SBSS satellite. Pass durations of about 20 min seemed to perform well, and could possibly be achievable in real operations. The 360 minute cycle time seemed to work acceptably, but shorter cycle times did have an improvement for the GPS and GEO SBSS locations.

7. OD Results – Space Based and Ground Based Observations

To obtain a sense of realism, I created approximate SSN locations to model a few US AF SSN deep space sensors. The simulator produced observations given time constraints for the sensors so they didn't receive continuous observations – just like the SBSS. Due to the geometry, there was not a common number of sensors that had visibility with each RSO satellite.

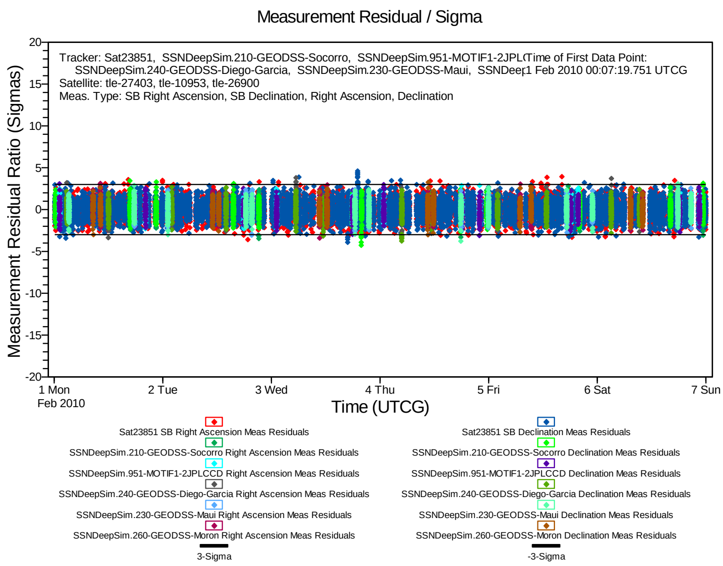

Figure 6. Filter Residual Ratios LEO SBSS tracker, GEO RSO satellites, and SSN. This plot shows the residual ratios for the SBSS scenario. This case has the exact initial satellite state vectors from simulated data with all errors turned on. There are 3 satellites modeled here and sensor observations from multiple SSN sites.

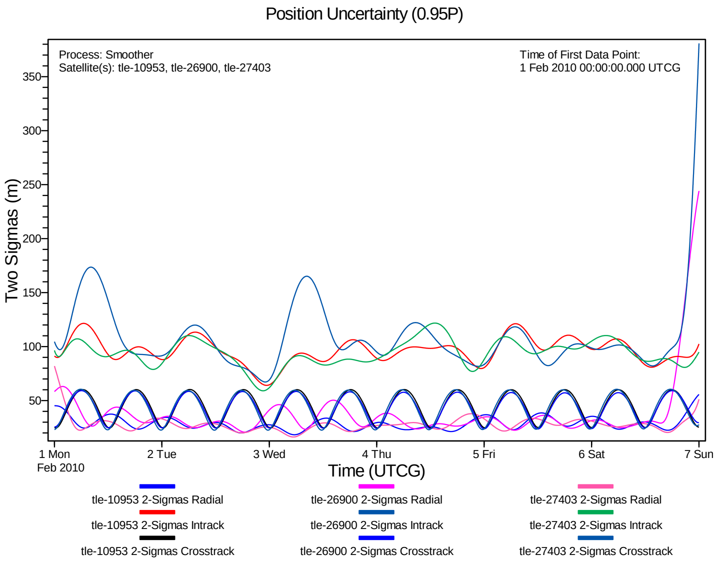

Figure 7. Smoother Position Uncertainty LEO SBSS tracker, GEO RSO satellite, and SSN. This plot shows the position uncertainties for the SBSS scenario. This case has the exact initial satellite state vectors from simulated data with all errors turned on. There are 3 satellites in the filter model here.

These tests resulted in the following results.

Table 11. Average Smoother Position Uncertainty with SBSS tracker and SSN. This table shows the position uncertainties for the SBSS scenario. This case has the exact initial satellite state vectors from simulated data with all errors. The number of SSN sensors that had visibility to the satellite is also included.

SBSS	Satellite	SBSS obs time 5	20	60	360	# SSN sensors	Avg Acc (m)	Cycle Time (min)
LEO	10953	x				3	263	360
LEO	26900	x				2	390	360
LEO	27403	x				1	318	360
LEO	10953		x			3	120	180
LEO	26900		x			2	123	180
LEO	27403		x			1	114	180
LEO	10953		x			3	111	360
LEO	26900		x			2	115	360
LEO	27403		x			1	111	360
LEO	10953		x			3	122	720
LEO	26900		x			2	145	720
LEO	27403		x			1	118	720
LEO	10953		x			3	115	1440
LEO	26900		x			2	117	1440
LEO	27403		x			1	113	1440
LEO	10953			x		3	40	360
LEO	26900			x		2	41	360
LEO	27403			x		1	41	360
LEO	10953				x	3	26	360
LEO	26900				x	2	27	360
LEO	27403				x	1	27	360

SBSS	Satellite	SBSS obs time 5	20	60	360	# SSN sensors	Avg Acc (m)	Cycle Time (min)
GPS	10953	x				3	461	360
GPS	26900	x				2	830	360
GPS	27403	x				1	1344	360
GPS	10953		x			3	256	180
GPS	26900		x			2	341	180
GPS	27403		x			1	329	180
GPS	10953		x			3	389	360
GPS	26900		x			2	625	360
GPS	27403		x			1	774	360
GPS	10953		x			3	524	720
GPS	26900		x			2	1221	720
GPS	27403		x			1	1561	720
GPS	10953		x			3	515	1440
GPS	26900		x			2	907	1440
GPS	27403		x			1	1496	1440
GPS	10953			x		3	494	360
GPS	26900			x		2	360	360
GPS	27403			x		1	357	360
GPS	10953				x	3	30	360
GPS	26900				x	2	38	360
GPS	27403				x	1	34	360

SBSS	Satellite	SBSS obs time 5	20	60	360	# SSN sensors	Avg Acc (m)	Cycle Time (min)
GEO	10953	x				3	450	360
GEO	26900	x				2	939	360
GEO	27403	x				1	1021	360
GEO	10953		x			3	101	180
GEO	26900		x			2	132	180
GEO	27403		x			1	125	180
GEO	10953		x			3	426	360
GEO	26900		x			2	875	360
GEO	27403		x			1	694	360
GEO	10953		x			3	453	720
GEO	26900		x			2	1067	720
GEO	27403		x			1	1272	720
GEO	10953		x			3	535	1440
GEO	26900		x			2	991	1440
GEO	27403		x			1	2328	1440
GEO	10953			x		3	129	360
GEO	26900			x		2	226	360
GEO	27403			x		1	166	360
GEO	10953				x	3	62	360
GEO	26900				x	2	63	360
GEO	27403				x	1	62	360

The results were very similar to the previous SBSS-only results. The last trials are probably about as good as can be expected because it results from near continuous observations. As the SBSS observations became more numerous, they often overwhelmed any benefit of the data fusion of the SSN contribution, although there was still some slight improvement in the results.

8. Combined Analysis – IOD and OD

Lastly, we examine the combined effect of IOD and OD for each SBSS location. The reduced initial accuracy due to the IOD process should influence the subsequent OD.

Beginning with the LEO SBSS, tle-10953, we process the IOD with 15 obs, sequential, 5 ER initial range, and 1.5 min observation spacing. The solution is

```
                sma (km)       ecc        incl (°)      raan (°)     argp (°)     arglat (°)
tle-10953   41917.7628   0.00435127   14.632729   358.698471   197.667717   26.506011
Epoch 1 Feb 2010 00:11:37.105 UTCG
```

Now, because we are using a less accurate initial state, the initial RIC uncertainties are changed from 50 / 100 / 20 and 0.06 / 0.04 / 0.02 to 3000 / 6000 / 2000 / 1.5 / 0.8 / 0.5 m and m/s. Running the filter and smoother with just SBSS observations, the results were not very good. This is expected because angles-only techniques do *not* (usually) result in a state that is accurate enough for a filter to process directly. A LS object was inserted. The revised state from a 4 hour LS process was as follows.

```
                sma  (km)         ecc          incl (°)       raan (°)     argp (°)      arglat (°)
tle-10953     42171.936929     0.00062854     14.430133     358.329019     1.408092     26.855525
                             Position vector (km)                        Velocity vector (km/s)
               38123.973538    17335.041407    4744.762649    -1.31077193    2.69751198    0.68398542
Epoch 1 Feb 2010 00:11:37.105 UTCG
```

The filter and smoother were run to assess the accuracy. Three satellites were simultaneously solved for the solution.

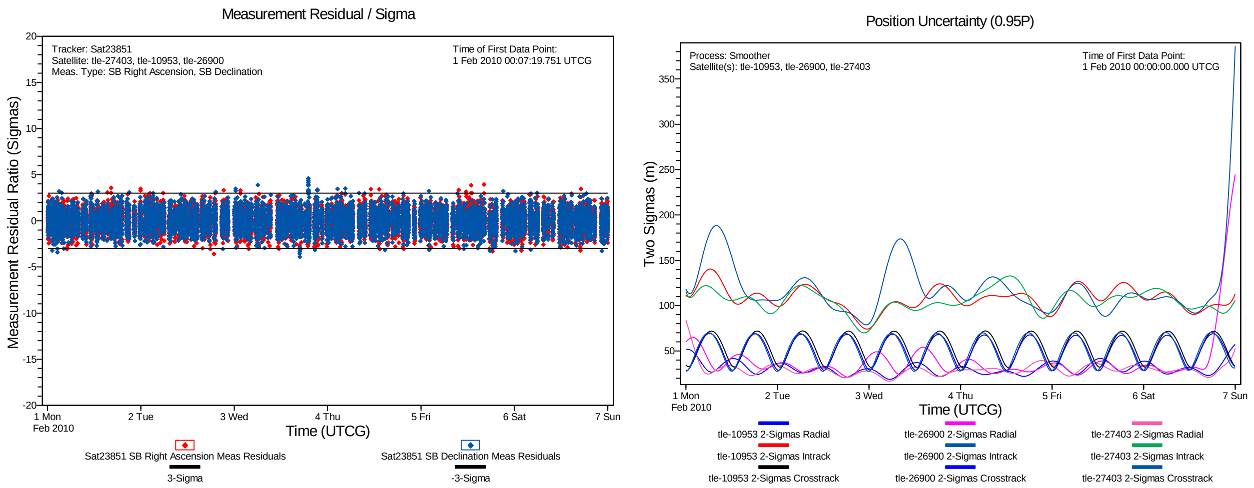

Figure 8. Residual Ratios and Smoother Position Uncertainty LEO SBSS tracker and GEO RSO satellites. These plots show the residual ratios and the position uncertainties for the SBSS scenario. The RSO state is derived from an IOD process and then a short LS run.

Now, inserting the ground based observations from the SSN sensors, we get the results in Fig. 9. Notice that the additional observations bring the overall accuracy down very little (the scales between Fig 8 and 9 are the same for the position uncertainty).

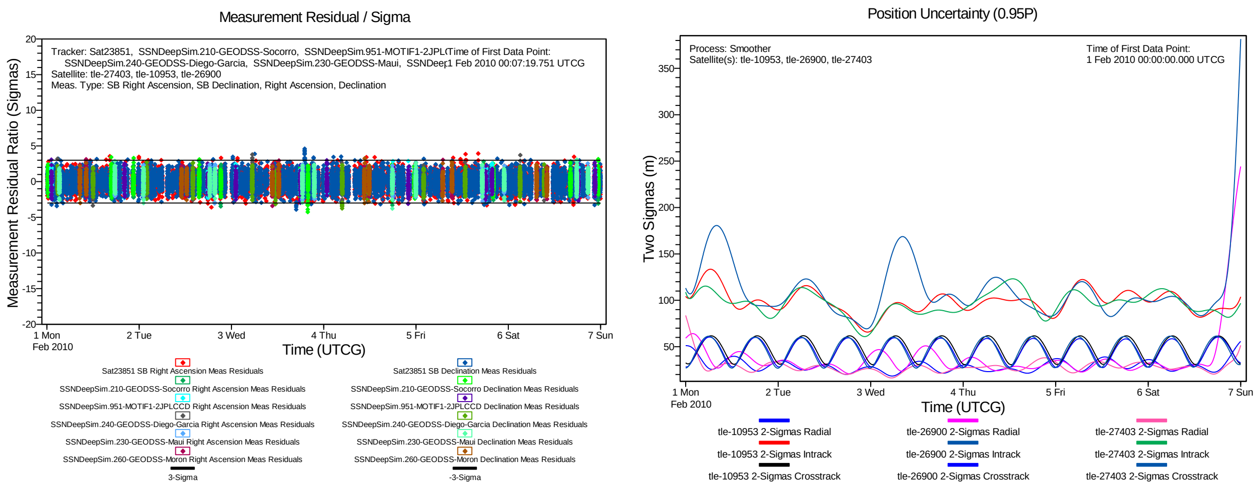

Figure 9. Residual Ratios and Smoother Position Uncertainty LEO SBSS and SSN trackers and GEO RSO satellites. These plots show the residual ratios and the position uncertainties for the SBSS scenario. The RSO state is derived from an IOD process and then a short LS run.

The results are the same as the results shown in Table 10. To effectively examine the results for the combined analysis, we must also specify how many satellites are in the solution. The filter/smoother can produce better results if multiple satellites are in the solution because the combined processing exploits any correlations and better estimates sensor characteristics. The previous results used solutions of 3 RSO satellites. Had we used only one, we would have seen results as in Fig. 10 below – also shown are the 5 satellite results (GPS SBSS location).

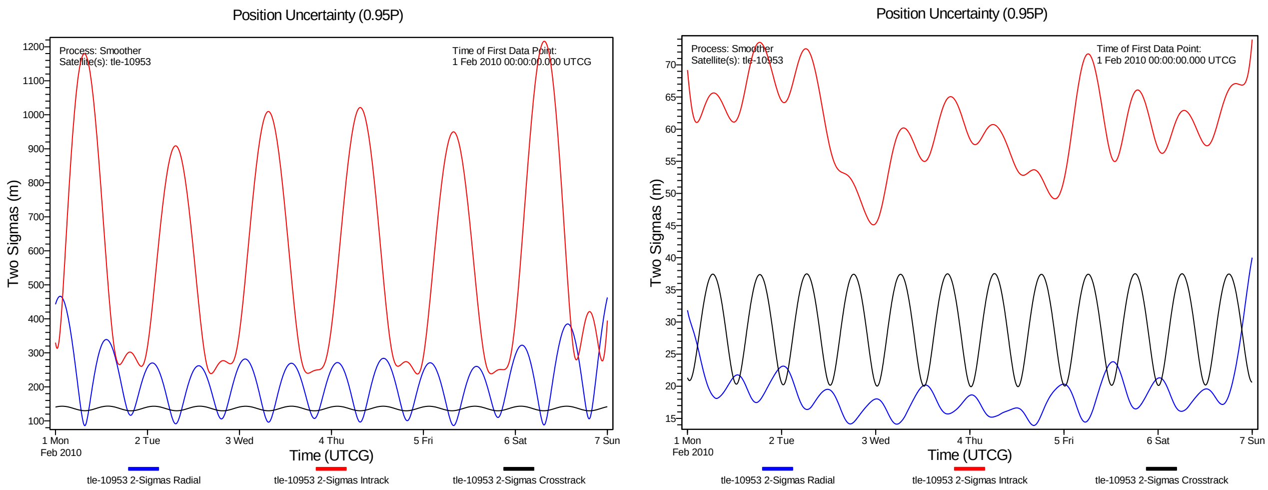

Figure 10. Smoother Position Uncertainty LEO SBSS tracker and GEO RSO satellite. The left hand plot shows the results for processing the single RSO satellite. The right hand plot shows the results when processing 5 RSO satellites. Notice the improvement in the multiple satellite solution. All RSO states are derived from an IOD process.

The filter and smoother are obtaining coupled information between the satellites in the solution. This lets the observation noise and bias be estimated better (more accurately), and the solution improves.

The GPS IOD was not close enough to the correct RSO answer for initial solution, and the case is under further study and will be presented at the Toronto conference this summer.

For the GEO SBSS, we find the following. Taking the GEO SBSS, tle-10953, we process the IOD with 15 obs, sequential, median, 5 ER initial range, and 0.5 min spacing, we find the new TLE-10953 orbit as

	sma (km)	ecc	incl (°)	raan (°)	argp (°)	arglat (°)
tle-10953	42162.690200	0.00026341	0.085249	95.394508	215.502660	345.879983
	6397.197841	41681.826693	-15.306523	-3.03851329	0.46696746	0.00443559
Epoch 1 Feb 2010 01:30:48.785 UTCG						

Now, because we are using a less accurate initial state, the initial RIC uncertainties are changed from 50 / 100 / 20 and 0.06 / 0.04 / 0.02 to 3000/6000/2000/0.9/0.6/0.3 m and m/s. The velocity RIC values were changed to reflect the different accuracy in the initial GEO orbit. The GEO cases seemed accurate enough from the IOD, so we run that case.

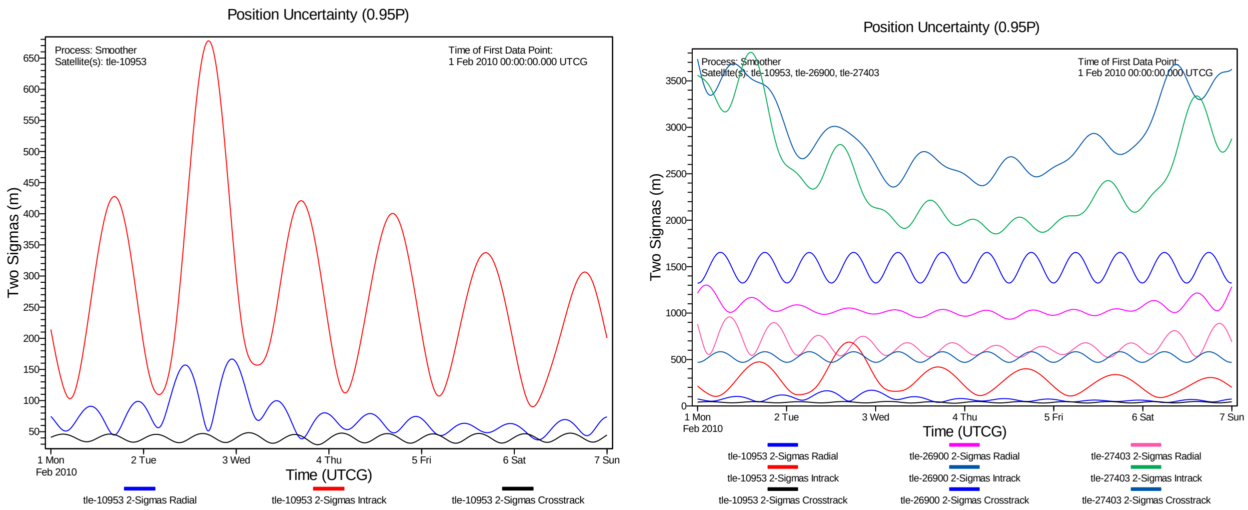

Figure 11. Smoother Position Uncertainty GEO SBSS tracker and GEO RSO satellites. The left hand plot shows the results for processing the single RSO satellite. The right hand plot shows the results when processing 3 RSO satellites. The initial satellite 10953, is the same in both plots. All RSO states are derived from an IOD process.

Including ground observations of the 3 satellites does not change the results substantially, indicating some degree of observability issues. This is under further investigation for the Toronto conference.

9. Discussion

The largest gains to GEO SSA seemed to come from fusing SB data with the ground based measurements. Continuous observations gave better results, but are not realistic in an operational sense. However, continuous observations *may* benefit monitoring the immediate region around a satellite. In this case, a staring sensor is likely much better as the continuous observations will detect any new objects entering or departing the region.

For the SBSS placement in LEO, the relative motion between the sensor and GEO RSO satellite is large, but the rapid orbital revolutions of the SBSS seemed to give reliable observations to support IOD and OD.

The GPS SBSS placement performs favorably for most of this analysis. The relative motion between the SBSS and the GEO RSO satellite is sufficient to be observable, there can be long dwell times (or short if needed), and the range is closer to support lower magnitude observation detections.

The GEO SBSS placement worked acceptably when fused with ground based data, but there were some severe observability problems when using solely SBSS observational data. Increased GEO accuracy for SSA may need to use additional GEO SBSS sensors to overcome this result. However, the proximity and ability to provide continuous observations make it more likely to succeed in proximity evaluations. Thus, it may be better for GEO SBSS sensors observations to be used in determining if objects are approaching or departing the local area.

Underlying all this is the ability of the sensor to track satellites, or for it to be fixed in space. Sharma et al (2002) discuss the geosynchronous pinch point and the capability of the sensor to stare at this location and track many satellites. Additional study is needed here.

10. Conclusions

Several options related to the placement and processing of a space based surveillance platform and its ability to improve the SSA at GEO were examined.

Two primary areas were studied – the IOD aspects, and the subsequent orbit determination. The conclusions are divided along those lines. For the IOD portion of testing:

a. Placement of the SBSS didn’t seem to matter too much, although as expected, the lower orbits enabled much more precise orbits for the SBSS vehicle, and sometimes for the GEO RSOs. There are obviously additional trades that need to be made, but the length of time for the sensors to observe the SBSS seem to be the most critical factor in determining what positional accuracy will be available.

b. Some of the cases analyzed were extremely difficult, if not exactly singular cases for angles-only solutions. As such, existing published techniques will not accurately process the orbits in a reliable fashion. This was especially apparent in the GEO SBSS placement.

c. The IOD method from Gooding appears to be quite robust. It solved a majority of the cases with the default settings, including near singular cases, although some cases proved inconclusive. The filter responded quite well to the various cases, but the observability prevented a significant amount of the data form ever being processed, so traditional “bathtub” performance was not always seen.

d. A single IOD result is generally not sufficient due to the variability of individual cases (ie, which points you pick). If you pick the wrong combination of observations, your answer could be a complete failure.

e. The best IOD results came from cases where the initial estimate of range (the single unknown in the problem) is closest to the actual value.

f. Averaging the orbital elements (non-fast variables) provided some benefit, but the mode (or median given sufficient trials – 10 to 20 for example) seemed to work a little better.

g. All permutations of the orbital elements appeared to yield better answers than just the sequential trails, and observations spaced farther apart (say about 1-2 minutes) seemed to do better than closely spaced observations.

For the OD portion of the tests, we found the following items.

a. Thoroughly understanding the particular geometry is extremely important in these tests. One could spend considerable time trying to “improve” the filter or smoother results from singular cases when in fact, the geometry simply prevents an accurate and reliable solution.

b. The best results occurred with the LEO and GPS based SBSS locations as this provided a greater amount of relative motion between the satellites – a necessary condition for determining the orbit with greater fidelity.

c. Although the SBSS observations were often sufficient to obtain orbital information, better accuracy occurred when the ground based SSN resources were included as well.

Several technical areas are under investigation for additional study, especially related to the combined processing.

11. REFERENCES

Gooding, R. H. 1990. A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem. Celestial Mechanics and Dynamical Astronomy. 48(2): 145-165.

Gooding, R. H. 1993. A New Procedure for Orbit Determination Based on Three Lines of Sight (Angles-only). Defense Research Agency DRA TR 93004. Farnborough, Hampshire.

Gooding, R. H. 1997. A New Procedure for the Solution for the Classical Problem of Minimal Orbit Determination from Three Lines of Sight. Celestial Mechanics and Dynamical Astronomy. 66(1): 387-423.

Moulton, Forest R. 1914. An Introduction to Celestial Mechanics. Dover Publications, NY.

Long, Anne C. et al. 1989. Goddard Trajectory Determination System (GTDS) Mathematical Theory (Revision 1). FDD/552-89/001 and CSC/TR-89/6001. Goddard Space Flight Center: National Aeronautics and Space Administration.

Sharma, Jayant, Grant H. Stokes, Curt von Braun, George Zollinger, and Andrew J. Wiseman. 2002. Toward Operational Space Based Space Surveillance. *Lincoln Laboratory Journal*. 13:2. pg 309-334.

Vallado, David A. 2007 *Fundamentals of Astrodynamics and Applications*. Third edition. Springer/Microcosm, Hawthorne, CA.