Effect of Radar Measurement Errors on Small Debris Orbit Prediction

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Abstract

This paper reviews the basic radar requirements for tracking small debris (1 to 10 cm). The frequency and sensitivity of current US radars collecting debris data are reviewed. An analysis of the tracking errors of these radars is provided. Based on their range, velocity, and angle track errors their capability to provide orbital period and inclination data on small debris is assessed. The current criteria to catalog space objects are reviewed in light of current radar collection data on small debris. Recommendations are made for radar parameters and filter criteria for cataloging small debris.

Introduction

It is estimated that there are more than 20,000 debris objects with diameters larger than 10 cm (and 600,000 with diameter larger than 1 cm) orbiting the earth (Ref: Air & Space, April 6, 2011) as a result of four decades of space activity. This estimate includes the functioning satellites, but by far the most objects are what are called space debris (SD), man-made orbital objects which no longer serve any useful purpose. Many of the small-sized (less than 10 cm) particles are due to explosions of spacecraft and rocket upper stages, but there are also exhaust particles from solid rocket motors, leaked cooling agents, and particles put into space intentionally for research purposes. The large (> 10 cm) objects have known orbits and are routinely monitored by the U.S. Space Surveillance Network, but information about the smaller particles is fragmentary and mainly statistical.

The current spacecraft shielding, such as that used for the manned modules of the Space Station, are only capable of protecting against debris with diameters below about 1 cm. As their only means of protection, Spacecraft maneuvering is required to avoid collision with debris larger than 1 cm. This, however, requires that the orbit of the debris object be precisely known. Currently the US Space Surveillance Radars used to track low earth orbit(ing), LEO, objects are only tracking debris like objects down to 5 cm. Of the estimated 600,000 objects above 1 cm, only some 22,000 can be tracked as of today. This leads to wide uncertainties in the estimated quantities of debris, and their predicted

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orbits. If a collision with larger debris does occur, many of the resulting fragments from the damaged spacecraft will become an additional collision risk.

Historically NASA has sponsored measurement and modeling efforts to characterize the LEO debris environment. The NASA Orbital Debris Program Office at Johnson Space Center has developed orbital debris engineering models to estimate the orbital debris environment (debris spatial density, flux, etc.).[1] Models such as ORDEM2000 provide a complete description of the environment in terms of debris flux onto spacecraft surfaces or debris detection rate observed by a ground-based sensor. Other models, such as the NASA SBRAM (satellite breakup risk-assessment model) and the NASA EVOLVE (long-term debris evolution model), are more applicable to evaluating the short-term collision risk, due to fragments from recent breakup events, and the long-term impact of various mitigation measures on the debris environment including secular effects such as the solar activity cycle, which affects atmospheric density and, hence,the decay rate of objects in low Earth orbit (LEO), the growth of the space vehicle population, and a projected fragmentation rate. It should be noted that the NASA models are not intended to catalog debris (i.e., create debris elements sets for avoidance), but rather to determine the LEO debris population for risk assessment and spacecraft design. The major data sources for the population are:

- The US Space Surveillance Network (UHF and VHF radars) catalog which builds the 1-m and 10-cm populations
- The MIT/LL Haystack (X-band) and HAX (Ku-band) radar data which build the 1-cm population, but do not catalog the population
- The LDEF (Long-Duration Exposure Facility) measurements which build the 10-microm and 100-microm populations

The United States Space Command Space Surveillance Network is composed of ground- and space-based sensor systems to track resident space objects. The FPS-85 located at Eglin Air Force Base supplies the majority of the element set and RCS data for LEO objects. The VHF NAVSPUR Fence contributes to the element set data base. These data are compiled daily into Keplerian element sets. In addition to the two-line element data, the radar cross section (RCS)-size data set and the area-to-mass ratio database are also maintained. The NASA Size Estimation Model (SEM) is used to derive size from RCS measurements.

Since there is only very limited element set data available for the 1 to 10 cm population, the ability to avoid debris in this population is extremely limited. In an effort to solve this problem the US Air Force awarded Lockeed Martin a $107 million Space Fence Contract this year to develop an S-band Space Fence to replace NAVSPUR. In addition the US has upgraded the software in the FPS-85 system to test a “Debris Fence”.[2] Since multi-sensor tracking data are a key element in verifying and updating new object catalog entries, a key issue is the capability of the current LEO radar systems to provide precision tracking data to help catalog the high risk 1 to 10 cm population in support of the potential new S-band Fence. The ability to improve the track quality of the current radar systems (particularly the range, velocity, and angle measurements) will determine their capability to provide precise orbital period and inclination data on small debris sufficient for cataloging.

Current Capability

In assessing the capability of current debris detection radars to generate orbits on space debris a number of issues will be addressed. These include determining the current radar sensitivity (i.e., the detectable RCS as a function of range) and track capability (i.e., track time, measurement errors). The improvement in track capability/accuracy that can be achieved by changing the operating modes of the current debris detection radars will be reviewed, including the reduction of range measurement errors to enhance orbit prediction and ultimately cataloging. Track accuracy is a function of the sensitivity (i.e., signal-to-noise ratio, SNR) of the radar on a given target. The sensitivity is a function of target size as measured by the Radar Cross Section (RCS) of the target. NASA has developed the Size Estimation Model (SEM) to estimate the size of the target based on the RCS.[3] The SEM is derived from multi-frequency measurements of thirty-nine "representative" debris objects selected from two hypervelocity impacts of simulated satellites and presents the size (estimated diameter) of the objects measured as a function of the measured RCS. Figure 1 shows the relationship for various frequencies, including those for the radars being reviewed. The approximation to the SEM results used here assumes a direct transition from the optical to the Rayleigh region.

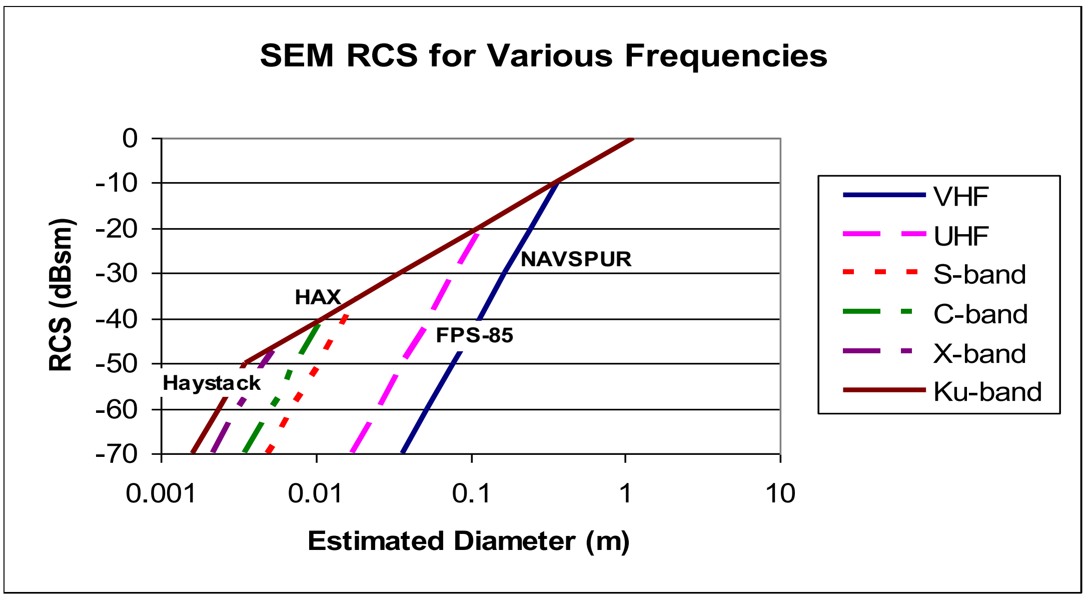

Figure 1. RCS Requirements to Detect Debris

The break in the curves differentiates between what nominally is called the Rayleigh region and Optical region. In the Optical region the RCS is independent of frequency. At the break the Rayleigh scattering varies as the sixth power of object size and the fourth power of wavelength. As shown, the FPS-85 operating at UHF requires extreme sensitivity to detect and track 1 to 5 cm objects. At the S-band frequency and higher the scattering of 1 to 10 cm objects is still in the Optical region, making the RCS sensitivity requirement significantly less than for lower frequencies. In this region RCS varies as the square of target (debris) size (estimated diameter). A 20 dB improvement in RCS sensitivity at a given range will result in a 10:1 reduction in the debris size that can be detected. Unfortunately, for the FPS-85 which is operating in the Rayleigh region for small debris the RCS varies as the 6^{th} power of size. A 60 dB improvement in RCS sensitivity is required to achieve a 10:1 reduction in detectable debris size. Thus, for the FPS-85 to detect a 1 cm target requires the radar to achieve a -70 dBsm sensitivity, while the X-band and Ku-band, Haystack and HAX, respectively, need only achieve a sensitivity to detect -40 dBsm targets. While the FPS-85 is limited due to its operating frequency, the question now is can improved operating modes (search/track at higher elevations, multiple pulse integration) improve sensitivity and track accuracy to allow the radar to track 1 to 5 cm debris? And can the Haystack and HAX radars, which currently operate with unmodulated pulses in a fixed beam staring mode, change their modes to allow target tracking? To determine the ability of each of the current debris measuring radars (FPS-85, Haystack and HAX) to meet these requirements the operating parameters of each will be reviewed.

Haystack/HAX

NASA has been using the MIT/LL Long Range Imaging Radar, known as Haystack, and the Haystack Auxiliary (HAX) Radar to characterize space debris in size, inclination and altitude since 1990. The HAX became operational in 1994 and is used primarily to observe the low earth orbit (LEO) debris environment. Although its sensitivity is lower than Haystack it has a wider field-of-view (1.7 times that of Haystack). The HAX observation mode is currently 75^o east. The average debris diameter detected has been reported as from 2 cm to several meters (based on the NASA Size Estimation Model, SEM). Haystack is reported to detect debris from less than 1 cm to several meters The Haystack/HAX debris detections are of limited quality to determine the particle's eccentricity accurately. These measurements represent statistical samplings of the population, and are, thus, subject to sampling error.

The Haystack antenna is a 36.6m parabolic reflector, the half power beamwidth is 0.056^o. The Haystack pointing accuracy is approximately 1.5 millidegrees. The slew rate of the antenna is 2^o/second. The slew rate acceleration is $1.8^o/\text{second}^2$. The narrow beamwidth and the slow slew rate acceleration are too slow to allow Haystack to provide stare and chase tracks on low SNR debris. A cued search would be required.

The HAX antenna is a 12.2m parabolic reflector, the half power beamwidth is 0.10^o. The HAX pointing accuracy is approximately 2 millidegrees. The slew rate of the antenna is 10^o/second. The larger beamwidth (smaller reflector) and higher slew rate of HAX radar might be able to support stare and chase tracks on low SNR debris.

Figure 2 is a summary of the Haystack measurements during the 2003 measurement campaign. The large population of debris between 850 and 1000km altitude has been identified as small spherical droplets of eutectic sodium-potassium (NaK) coolant. The NaK coolant leaked from fast neutron reactors that separated from the Russian Radar

Ocean Reconnaissance Satellites (RORSATs) at the end of their lifetime. Estimates based on the Haystack measurements indicate that the majority of these objects have an estimated size less than 2 cm. Also the presence of a near-circular debris ring in polar orbit in the 1200 to 1400km altitude region has been observed. Most of the debris objects in this ring are less than 4 cm. The altitude, inclination and observation times of the

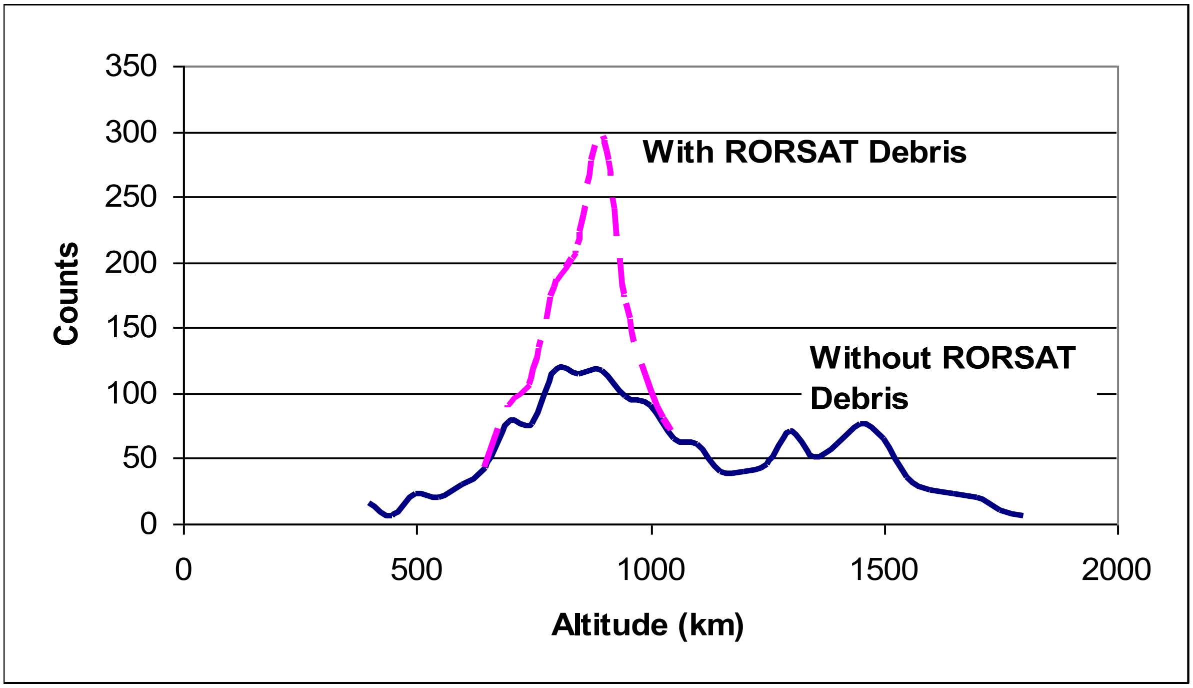

Figure 2. Haystack FY 2003 Collection

debris correspond to the orbit plane of the nuclear powered SNAPSHOT satellite which is well know for shedding pieces of debris (more than 50 pieces have been cataloged). Thus not only must the radars meet the RCS requirements of Fig.1, they must also do so primarily over the altitude region from 500 km to 1200 km as depicted in Fig 2.

FPS-85

The FPS-85 Phased Array Space Surveillance Radar, operational in 1969, is the only US phased array radar dedicated to space surveillance. The radar collects 16 million satellite observations per year. It can detect, track and identify up to 200 space objects simultaneously. It is the only phased array radar capable of tracking deep space objects (can track a basketball size object at 22,000nm). The bore-sight is at 45^{o}, the nominal low elevation surveillance fence is at 20^{o} elevation. The FPS-85 has upgraded software (1999) to erect a high elevation "debris" fence.[2] Developmental testing of a fence at 35^{o} enabled detection of objects greater than -35dBsm.

Radar Parameters

A summary of the radar parameters is given in Table 1. The Haystack/HAX parameters are those reported in the NASA report which summarizes the most recent debris collection campaigns.[3] When operating in this mode the radars used an

Table 1. Radar Parameters (In Debris Collection Modes)

Radar Parameter	FPS-85 (Trans/REC)	Haystack	HAX
Peak Power (kW)	32000	250	50
Frequency (GHz)	0.442	10	16.7
Beamwidth (deg)	1.3/0.7	0.058	0.10
Antenna Gain (dB)	43/48	64	67
Available LFM BW (GHz)	0.001	1	2
Pulse Width (msec)	0.25	1.64	1.64
Single Pulse SNR on 0 dBsm @ 1000km (dB)	64	59.2	40.6

unmodulated CW Pulse of 1.64 msec. with 16 pulse integration. Since the range measurement error is a function of pulse width, range accuracy in this mode is in the tens of kilometers.

The radar transmitters do have a high range resolution linear frequency modulation (LFM) mode, 1 MHz bandwidth for Haystack and 2 MHz bandwidth, which would significantly increase the range accuracy.[4] In the CW pulse mode it will be assumed that range-rate (Doppler) measurements were made. These measurements can yield velocity measurements which can be used to estimate orbit inclination more accurately than range-time processing with the very long pulses.

Parameters for the FPS-85 are partially available in Fact Sheets[5], articles[6] and published reports on the "Debris Fence"[7]. Based on the number of elements in the transmitter (5928) array and receiver array (19500) the antenna gains and beamwidths shown in Table 1 were computed using the standard equations from Skolnik[8]. Each element in the transmitter array is driven by a dedicated 0.25msec pulse width (4 kHz equivalent transmit bandwidth) radar transmitter unit and linear frequency modulation, LFM, (pulse chirping) of up to 1 MHz can be applied to enhance signal processing[9]. The single pulse SNR at 1000km was computed using the standard range equation[10]. A 4 kHz bandwidth, a loss of 6 dB, and receiver noise temperature of 300^o K were assumed in the calculation.

Predicted Radar Performance

The radar performance for each of the current debris collection radars in terms of the detectable debris as a function of altitude was computed using the SNR values at 1000 km as shown in Table 1. At each altitude the range to that altitude was computed and the SNR scaled to that range using the range to the 4^{th} power scaling from the standard range equation. For the Haystack and HAX radars the range was computed for an elevation angle of 75°. For the FPS-85 an elevation of 45° was assumed. This is consistent with raising the current search fence from 25° on boresight to near a boresight "Debris Fence". A SNR detection threshold of 10 dB was assumed in the calculation. This will result in a high probability of detection, but with a fairly high probability of false alarm. Designing a system to track all small targets will allow for multi-pulse processing which should mitigate the potential false alarm problem.

The results of scaling the SNR values to determine the detectable RCS at various altitudes are shown in Figure 3. The estimated diameter values shown in the figure were determined from the RCS using the SEM model depicted in Figure 1. The sensitivity of the Haystack and FPS-85 radars result in detectable RCS values in the Rayleigh SEM region. The sensitivity of the HAX, results in RCS values primarily in the optical SEM region. Thus, a slight increase in HAX sensitivity (i.e., using multi-pulse integration) can significantly increase the detectable debris size. In contrast, using multi-pulse integration in the FPS-85 will yield only marginal increases in the detectable debris size.

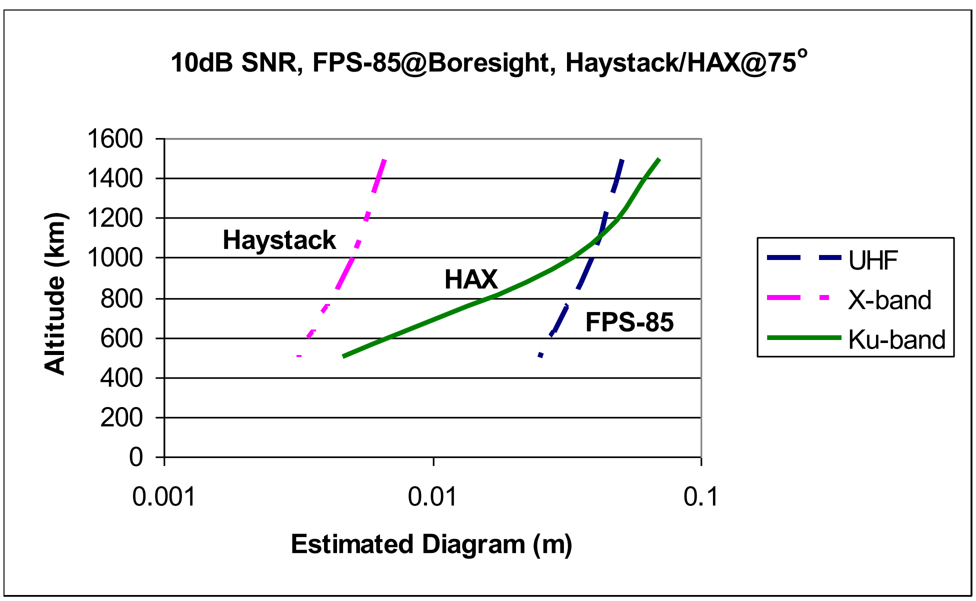

Figure 3. Predicted Radar Performance – Single Pulse

As shown, at maximum sensitivity the FPS-85 can detect a 2.5 cm estimated diameter debris object at 500 km. This is equivalent to detecting a -60 dBsm target at about 516 km with an SNR of 10 dB. At 1000 km a 3.9 cm piece of debris can be detected, which is equivalent to detecting a -49 dBsm target. A 12 dB loss in sensitivity is thus equal to a loss of 1.5 cm in detectable debris size. In comparison a 12 dB loss in HAX sensitivity will result in a change of detectable debris size of from 0.8 cm at 500 km to 3.4 cm. Clearly improving HAX sensitivity by integration or other means has a significant impact on its capability in the debris size region of interest.

Radar Measurement Errors

The precision with which a set of radar measurements, taken while tracking space debris, can produce an orbit for the debris will depend on the accuracy with which the measurements are made. The radars being evaluated here measure range, angle and velocity. The errors associated with these measurements must be determined before the radars ability to establish an orbit can be assessed.

The radar range measurement error, σ_r, is generally defined as the root-sum-square of three error components[11]

$$\sigma_R = (\sigma_{RN}^2 + \sigma_{RF}^2 + \sigma_{RB}^2)^{1/2} \qquad (1)$$

where the noise range error, $\sigma_{RN} = \Delta R/(2(SNR))^{1/2}$, ΔR is the radar range resolution, which is equal to c (the velocity of light) divided by twice the radar bandwidth; σ_{RF} is the fixed random error due to random noise in the receiver and is equivalent to the noise range error at a SNR of 20 dB; and σ_{RB} is the range bias error, since these errors are the same over a series of track pulses they will not affect track capability being assessed here.

The range resolution is defined by the pulse width in an unmodulated pulse system and is defined by the bandwidth in a modulated or pulse compression (e.g., LFM) system.

As in the case of range measurements, the measurement accuracy in each angular coordinate is characterized by the rms error, σ_A, given by the rss of the three error components;

$$\sigma_A = (\sigma_{AN}^2 + \sigma_{AF}^2 + \sigma_{AB}^2)^{1/2} \qquad (2)$$

where the noise angle error, $\sigma_{AN} = \theta/1.6(2(SNR))^{1/2}$, θ is the radar beamwidth, the factor 1.6 is derived from monopulse angle measurements ; σ_{AF} is the fixed random error which will limit angular accuracy for large values of SNR, due to random noise in the receiver angular errors will be assumed limited to $1/50^{th}$ of the beamwidth; and σ_{AB} is the bias error which will not affect short tracks.

Target radial velocity may be measured in one of two ways; either from multiple range measurements or from direct Doppler frequency measurements. The Doppler process will almost always result in better accuracy. The Doppler radial-velocity measurement accuracy is characterized by the rms measurement error, σ_A, given by the rss of the three error components;

$$\sigma_V = (\sigma_{VN}^2 + \sigma_{VF}^2 + \sigma_{VB}^2)^{1/2} \qquad (3)$$

where the noise velocity error, $\sigma_{VN} = \lambda/2\ \tau(2(SNR))^{1/2}$, λ is the wavelength and τ is the duration of the processed waveform; σ_{VF} is the fixed error and like the fixed range error case will be assumed limited to the noise error at 20 dB SNR; as with the range and angular errors, the bias error will not be considered for the tracking cases analyzed here.

The radar measurement errors are summarized in Table 2. The LFM range errors were computed for the maximum compressed pulse widths. In the case of the Haystack/HAX radars it is reasonable to assume that, in a debris tracking mode with 10 dB SNR and not a high resolution imaging mode, the limiting range error would be in the order of a meter (as opposed to the 1 to 3 cm error computed, assuming 2 to 1 GHz bandwidths). The velocity errors were computed for a single pulse, assuming Doppler processing. In the debris fixed beam detection mode Haystack has demonstrated the ability to use range-rate measurements to establish estimates of debris inclination[3]. The FPS-85 has the capability to transmit a LFM waveform. As a result it is reasonable to assume the enhanced error accuracy achieved using this waveform would be used in measurements intended to determine a debris orbit.

Table 2. Radar Measurement Errors

	Noise Error at Max Sensitivity (SNR 10 dB)			Fixed Error at SNR 20 dB & 1/50th Beamwidth		
	Range Error (km)	Velocity Error (m/s)	Angle Error (deg)	Range Error (km)	Velocity Error (m/s)	Angle Error (deg)
FPS-85						
Pulse CW	8.385	-	0.18	2.651	-	0.036
LFM (max)	0.033			0.011		
Haystack						
Pulse CW	54.950	2.0	0.008	17	0.65	0.001
LFM (max)	0.00003			0.00001		
HAX						
Pulse CW	54.950	1.3	0.014	17	0.41	0.002
LFM (max)	0.00002			0.000005		

Orbital Element Errors

Of the six orbital elements of a piece of orbiting debris, three are most accurately determined by radar measurements. These include the inclination, *i,* of the debris orbit to the plane of the equator, the longitude of the ascending node, Ω, and the orbital period, *T*. The approximate relationships for the 1 sigma errors in these three coordinates, δ_i (deg), δ_Ω (deg), δ_T (min) have the form[12]

$$\delta_i = 0.0123(R\ \sigma_A\ \pi/180) + 9.6(R\ \sigma_R\ /t_T^2) \qquad (4)$$

$$\delta_\Omega = 0.0123(R\ \sigma_A\ \pi/180) + 9.6(R\ \sigma_R\ /t_T^2) \qquad (5)$$

$$\delta_T = 0.025(R\ \sigma_A\ \pi/180) + 48(R\ \sigma_R\ /t_T^2) \qquad (6)$$

where R (km) is the radar range to the target, t_T (sec) is the track time, σ_R (km) is the sigma radar range error, and σ_A (deg) is the sigma radar angular error. These estimates seem to be based on empirical data and assume that the target altitude is less than 3000 km, the target eccentricity is less than 0.1 and the radar range is less 2000 km. In the relationships used here it is also assumed that the orbit inclination is greater than 60^o.

These relationships assume that only range and angle are measured during the track and not range-rate. If range-rate is measured the above relationships become;

$$\delta_i = \delta_\Omega = 0.0123(R\sigma_A\ \pi/180) + 1.2\ (R\sigma_V\ /t_T) \qquad (7)$$

$$\delta_T = 0.025(R\sigma_A\ \pi/180) + 6(R\sigma_V\ /t_T) + 6(R_{DOT}\ \sigma_V) \quad (8)$$

where R_{DOT} (km/sec) is the target range rate, σ_V (km/sec) is the sigma radar velocity error.

While these relationships are simple estimates for a rather complex problem, they are adequate to access the basic capability of the current radars to generate track measurements to predict debris orbits. The range, angle only relationships will be used to assess the orbit prediction capability of the FPS-85, since this is the normal operation of the radar. The range-rate relationships will be used to the assess the capability of the Haystack and HAX radars since in the debris collection mode (Pulse CW) the most accurate data would be obtained using range-rate measurements.

Track Time

From a review of the orbit prediction relationships it is apparent that radar track time will play a major role in establishing the prediction capability of the radars. The track time available for the Haystack and HAX radars in their fixed beam debris collection mode is a function of their beamwidths. For example, a piece of debris orbiting at 500 km in a circular orbit will have an average speed of 0.066 radians/minute. For the Haystack radar with a beamwidth of 0.01° this equates to a track time of 1.59 sec. For an orbit which passes directly through the beam this is 1.59 sec from a point 3 dB below the peak of the beam on one side to a point 3 dB below the peak on the other side. Thus there will be an effective loss in sensitivity during this equivalent track time. Using the average orbital speeds at other altitudes the Haystack and HAX track times were computed. The results are shown in Figure 4. The times shown are independent of the debris size. The minimum detectable size at each track time will be determined by the debris altitude. All larger size debris at this altitude will have the same track time.

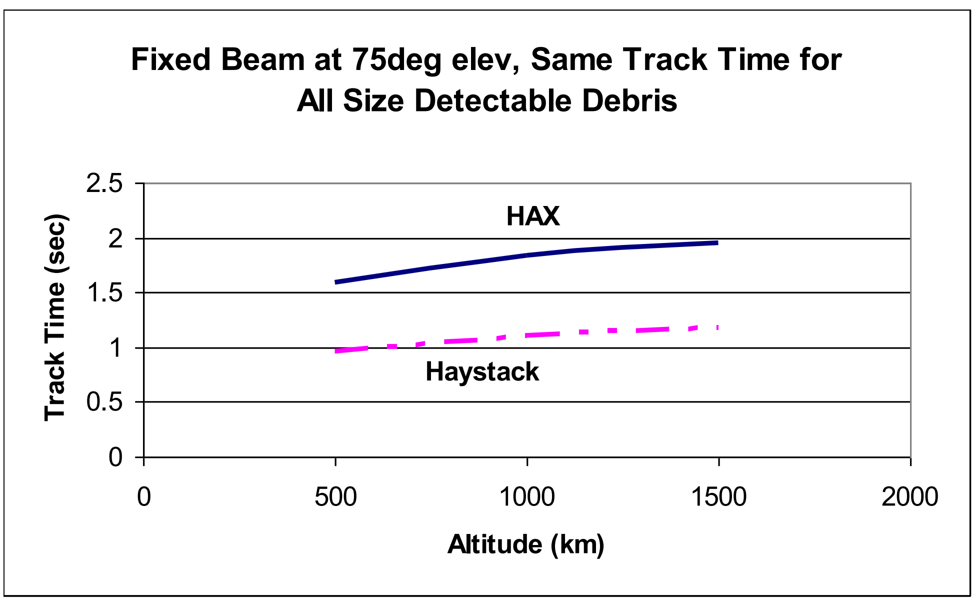

Figure 4. Available Haystack and HAX Track Time

The available track time for the FPS-85 array radar is more difficult to determine. Each orbit path through the radar's surveillance fence will result in a different track length from the point of entry in the fence to the point of exit from the radar's track field of view (FOV). A continuum of passes for a given orbit could be used to establish minimum and maximum track times available for each orbit. For this assessment it is more important to establish the effect of operating modes (i.e., fence elevation, pulse modulation, etc.) on orbit prediction capability than to assess the optimum performance on any particular orbit. As a result a simple spreadsheet simulation was used to compute the track time available on a single orbit at various altitudes. A 70° circular orbit was selected. The orbit was positioned to enter the surveillance fence on an ascending pass and exit the track FOV such that it passed through the array boresight at 45°. The range to the target at points along the trajectory through the FOV were computed and the equivalent detectable target RCS (and equivalent SEM diameter) determined. The results are shown in Figure 5 for two fence elevations. The 25° fence is the normal surveillance fence, which extends from 20° elevation at the edges of the azimuth FOV to 25° elevation at boresight azimuth. The higher fence was positioned to maximize sensitivity, but will provide less coverage. Note that the higher fence does decrease the detectable size at this

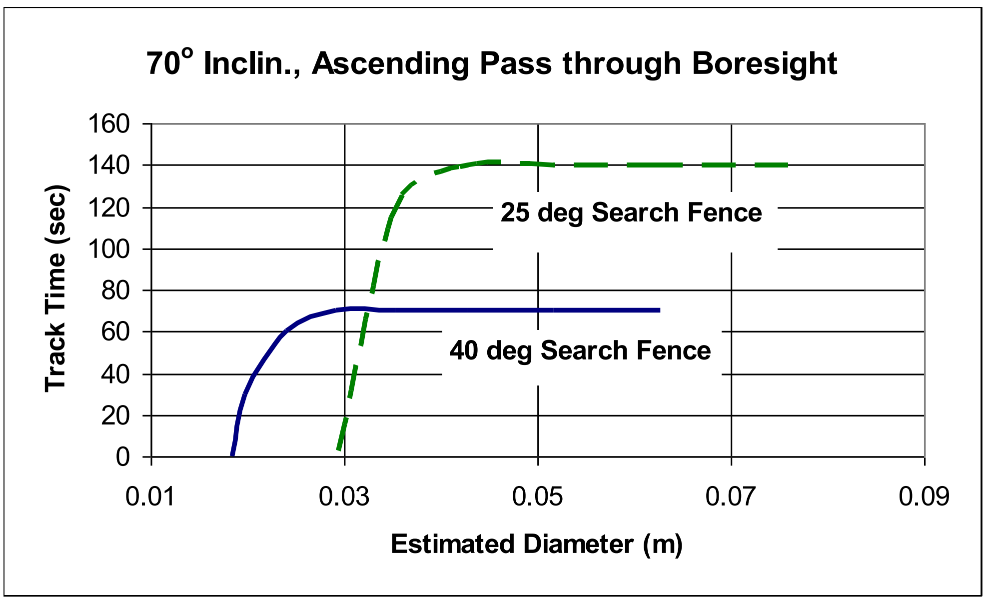

Figure 5. Available FPS-85 Track Time at 500 km Altitude

altitude, but at the expense of track time. The smaller track FOV limits the maximum time to about 70 sec., while the lower fence with less sensitivity at entry doubles the track time.

The results at 1000 km are shown in Figure 6. As was the case at the lower altitude the estimated detectable size has increased for the higher fence location, but at a two to one decrease in track time. Still a track time over 2 minutes generally results in good prediction accuracies.

The constant track time for large debris is a result of the single orbit used in this analysis. The total time is from entry in the surveillance fence to exit of the track FOV. This is the constant value shown (e.g., 325 sec in the case of the normal search fence).

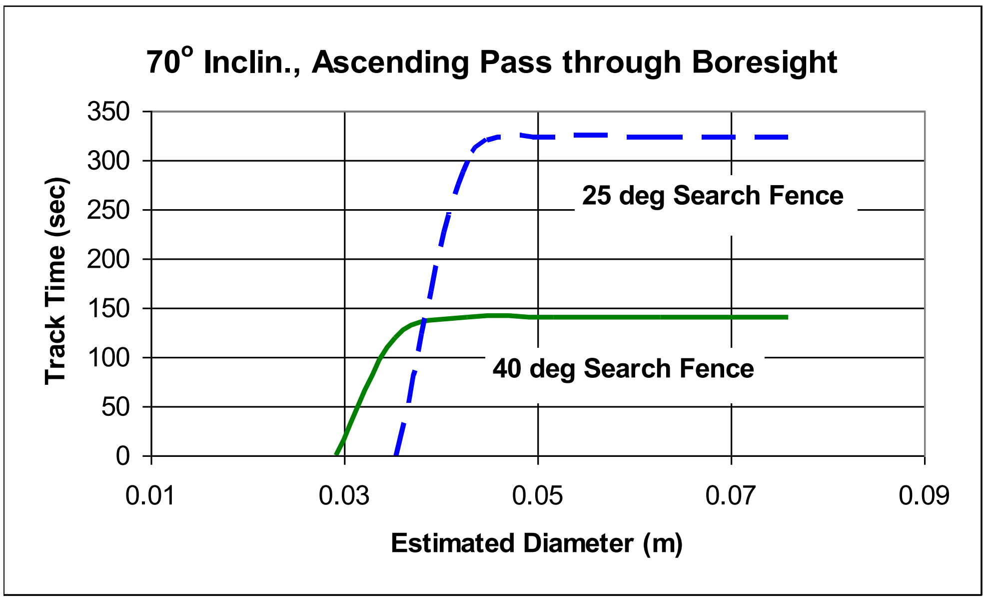

Figure 6. Available FPS-85 Track Time at 500 km Altitude

Orbit Prediction Error

The basic orbit elements available through radar measurements can now be assessed and their errors determined.

Period Error

Using the relationship given in (6) the period error was computed for the FPS-85 for a 70º circular orbit at 500 km. For the 40º elevation debris surveillance fence, the results are shown in Figure 7.

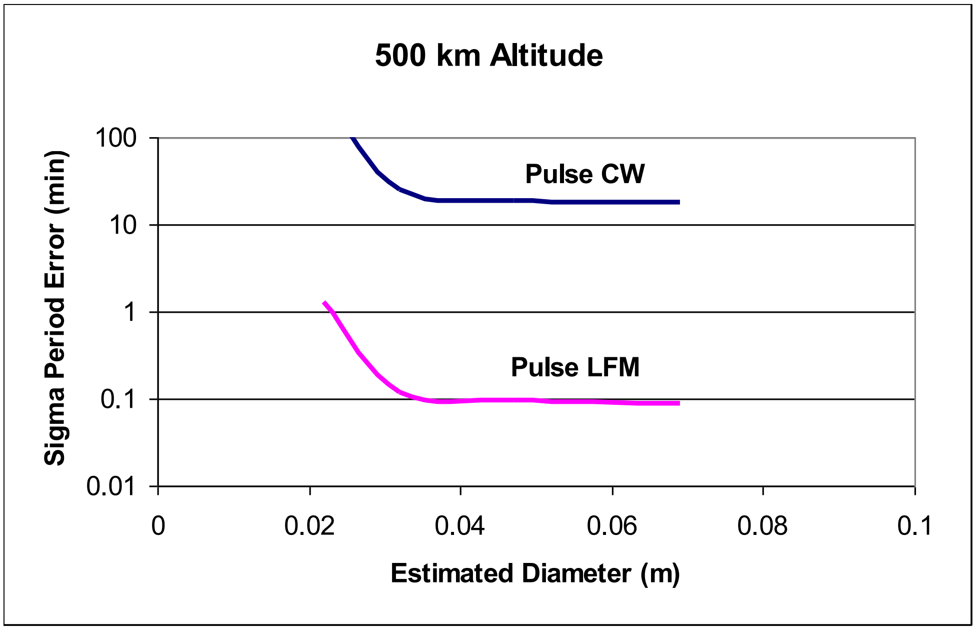

Figure 7. FPS-85 Period Error – 40º Fence, 500km Altitude

Two cases are shown. One in which a 250 ms pulse CW waveform is used and the second in which a 250 ms pulse LFM (1 MHz) waveform is used. The compressed waveform provides much better range measurement accuracy resulting in the roughly two order of magnitude improvement in predicted period accuracy.

As a result of this analysis the use of the pulse LFM waveform will be considered as the preferred debris tracking mode for this radar. This operating mode will be used to assess the track capability of the radar. The constant period error for large size debris is the result of the fixed range and angle errors for SNR sensitivity greater than 20 dB (see Table 2).

A comparison of the high elevation surveillance fence and the normal FPS-85 surveillance fence is shown in Figure 8. The lower 20º fence provides longer track times resulting in a lower period error, as shown. The higher fence provides better sensitivity and the ability to track smaller debris targets. At issue is whether the track accuracy for the higher fence is adequate to allow cataloging. If it is, then use of the higher fence is justified on the basis of the added sensitivity provided.

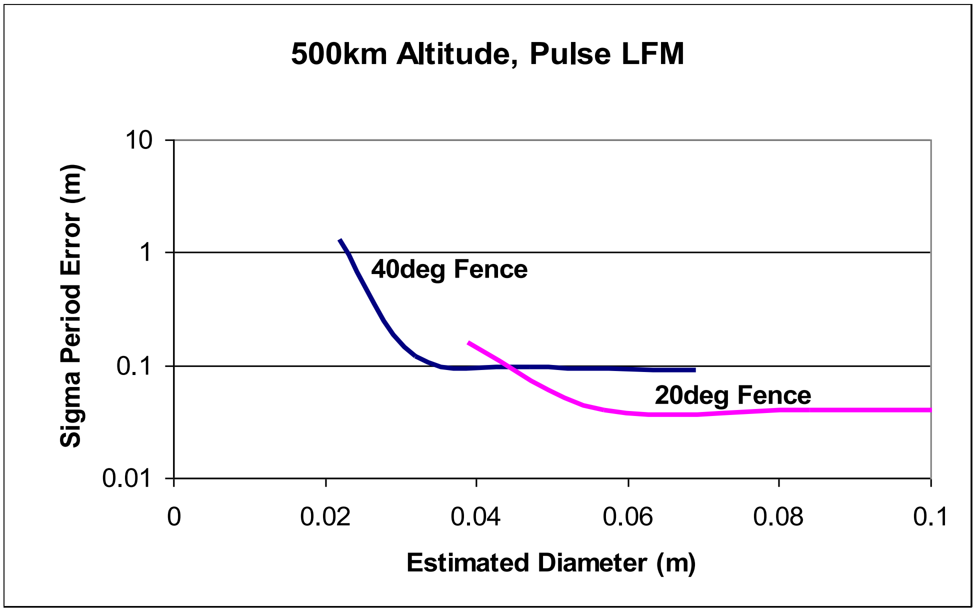

Figure 8. FPS-85 Period Error – Low Altitude Fence Comparison

The FPS-85 period error was also computed for a 1000km altitude. These results are shown in Figure 9. The effect of the longer track times at the higher altitude basically

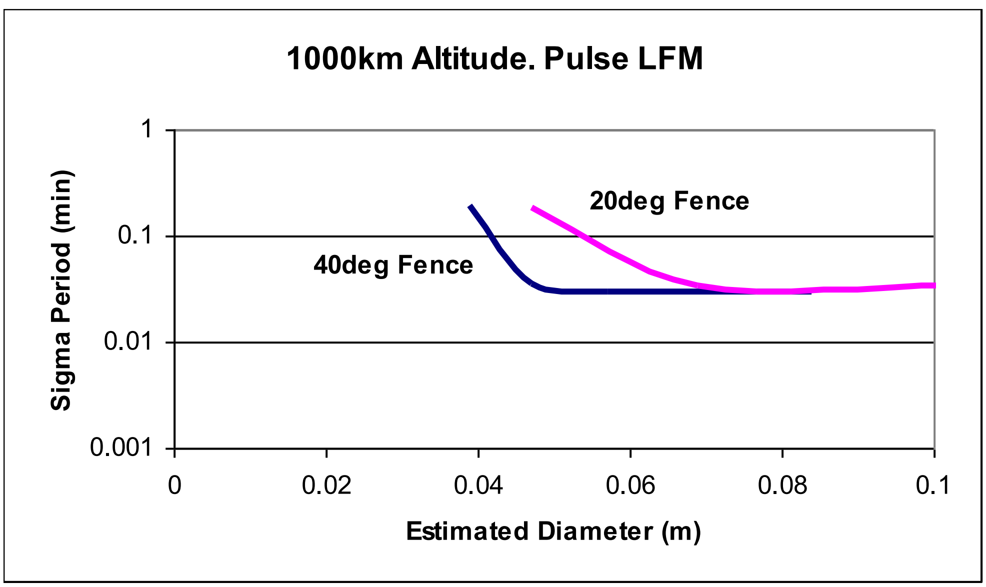

Figure 9. FPS-85 Period Error – High Altitude Fence Comparison

negates the range error term in (4) and the error is now only a function of the fixed angle and range resulting in the nearly equal constant error for debris sizes larger than the minimum detectable.

The period error for the Haystack and HAX radars was first computed assuming a pulse CW waveform operating mode, similar to the one used in the NASA debris collection measurement campaign. In this mode range-rate measurements are much more accurate than range measurements. As a result the relationship for range-rate measurements (8) was used in the calculations. The results for Haystack and HAX are shown in Figure 10 for an altitude of 1000 km. The slightly longer track time and better range-rate (velocity) accuracy of the HAX provides the low period error (1.6 min) shown. A general rule of thumb for orbit prediction accuracy is that the errors should be less than 1%. Haystack in the Pulse CW mode clearly does not meet this level (representing an almost 20% error). HAX provides marginal accuracy in this mode.

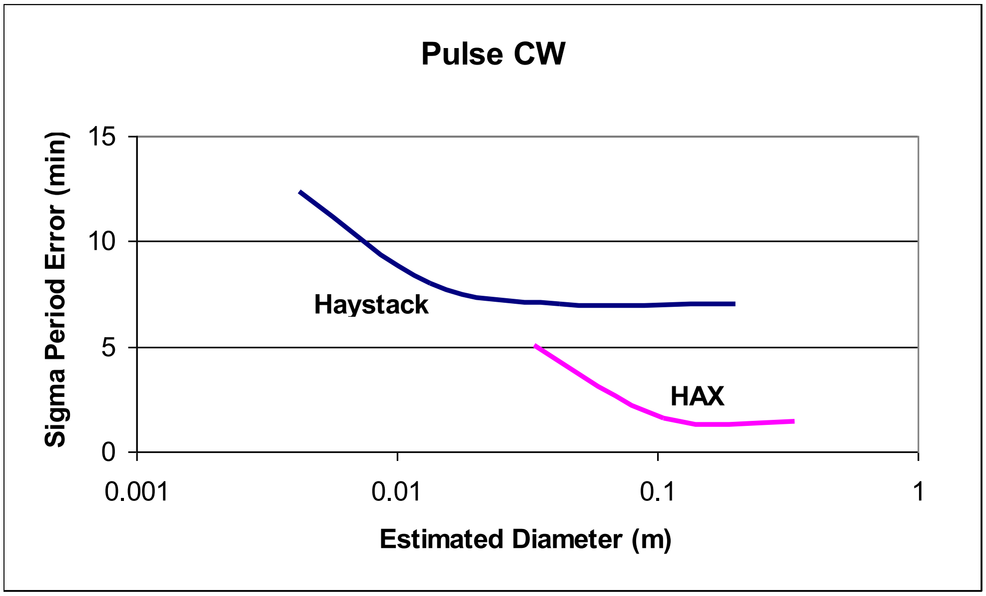

Figure 10. Haystack and HAX period error at 1000 km Altitude

The pulse CW mode clearly has its limitations in terms of track measurement accuracies. But, since the mode was used for the detection of small debris with a fixed beam, the mode selection is understandable. The single pulse HAX performance still does not provide the sensitivity required in the 1 to 10 cm region of interest. In an effort to look for alternatives to improve period accuracy an LFM waveform was investigated. As noted in the Parameter Table (Table 1) both Haystack and HAX has the capability to transmit and process LFM pulse compression waveforms. As noted earlier, the full bandwidth capability, while necessary for imaging, is not required to achieve accurate range measurements. As an example, a 7 MHz LFM waveform would produce a sigma range error of about 5 m. Considering a 1 millisecond transmit waveform this represents about a 7000:1 compression ratio. These are reasonable parameters to suggest using in a Haystack and HAX debris tracking mode.

It is unlikely that Haystack would ever attempt to operate in a stare and chase surveillance mode given the size of the antenna and its slew rate capability. It could, however, operate in a cued search mode, given a crude element set. In this mode the LFM track mode would be beneficial as well. HAX is much more likely to be able to operate in a stare and chase mode. In this mode it is reasonable to assume that a minimum 30 sec. track time could be obtained and the LFM mode proposed would be an adequate track mode to implement.

To improve HAX sensitivity pulse integration was considered. Figure 11 shows the net effect of a 7 MHz LFM waveform, 30 sec. track time, and 8 pulse integration on sensitivity and period prediction accuracy at an altitude of 500 km. The pulse integration improved the sensitivity to allow detection of 1 cm debris. The improved range accuracy

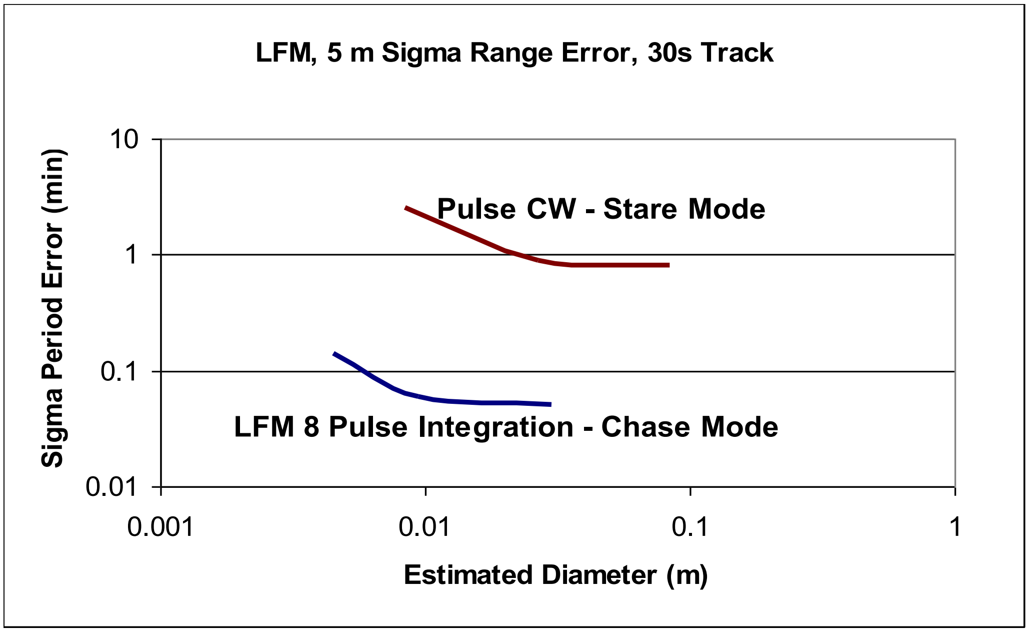

Figure 11. Comparison of HAX Period Error – 500 km

and 30 sec track time assumed in the chase (track) mode provides the improved period accuracy shown.

To be effective in supporting development of an element set catalog for small debris (1 to 10 cm) both Haystack and HAX should operate in a track mode consistent with the example used here. Whether in a cued search mode to provide updates of orbital data or in a possible stare-chase mode (HAX) both could be effective contributors, even in a part time role.

The effectiveness of the radars in this proposed LFM track mode is shown in Figure 12 for debris at 1000 km. Since track time and range error is the same for both radars they both achieve the same period error for target sizes larger than their minimum sensitivity. It would seem that the 8 pulse integration for HAX is not sufficient to provide the sensitivity desired in the 1 to 5 cm region. However, it should be noted that even at 1 cm the period error is still only 0.3 min. This may still be adequate for catalog maintenance.

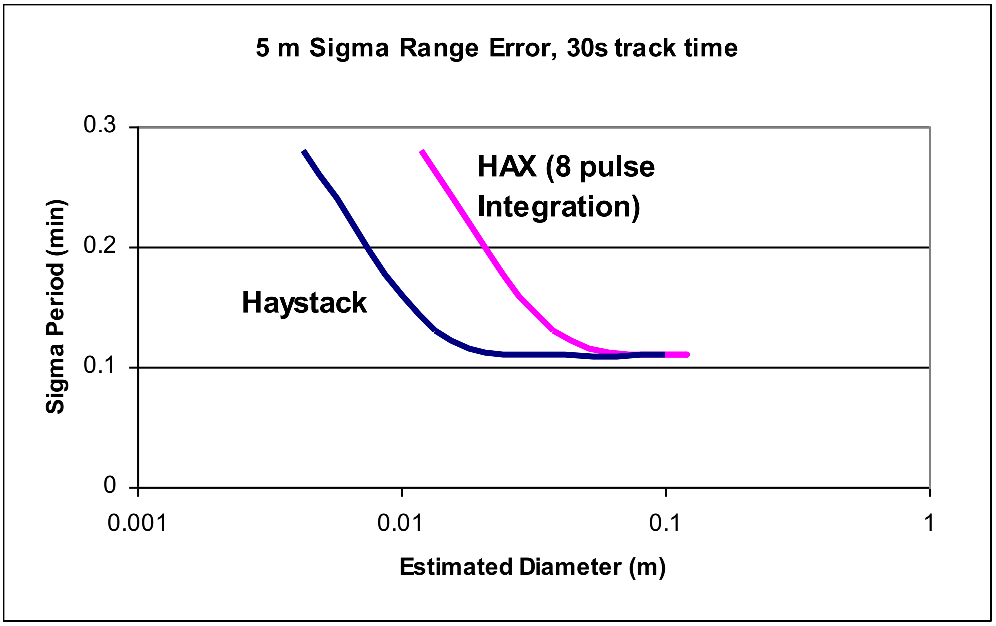

Figure 12. Haystack and HAX LFM Period Error at 1000 km

Inclination/Node Error

The inclination error was computed for the three radars using the same approach as used to compute the period error. Since the node and inclination are considered equal here, no distinction is made between node and inclination errors. Using the relationship given in (4) the inclination error was computed for the FPS-85 for a 70° circular orbit at 500 km. The results are shown in Figure 13 for both a 40° elevation debris surveillance

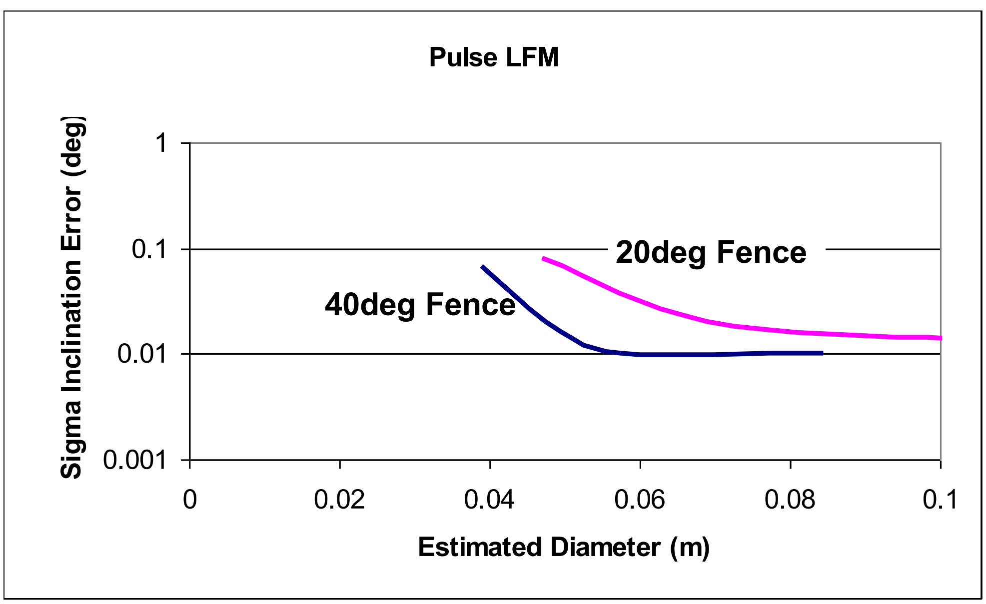

Figure 13. FPS-85 Inclination Error – 1000 km

fence and the normal 20° surveillance fence. A 1 MHz LFM transmit mode was used in the calculation consistent with the proposed operating mode for this radar.

The lower 20° fence provides longer track times resulting in a lower period error, as shown. The higher fence provides better sensitivity and the ability to track smaller debris targets. At issue is whether the track accuracy for the higher fence is adequate to allow cataloging. If it is, then use of the higher fence is justified on the basis of the added sensitivity provided.

The inclination error for Haystack and HAX was first computed using the Pulse CW mode employed in the NASA debris collection campaign. In this mode range-rate (Doppler) measurements provide the better track accuracy. As a result the relationship (7) was used to compute the inclination. The single pulse CW results are shown in

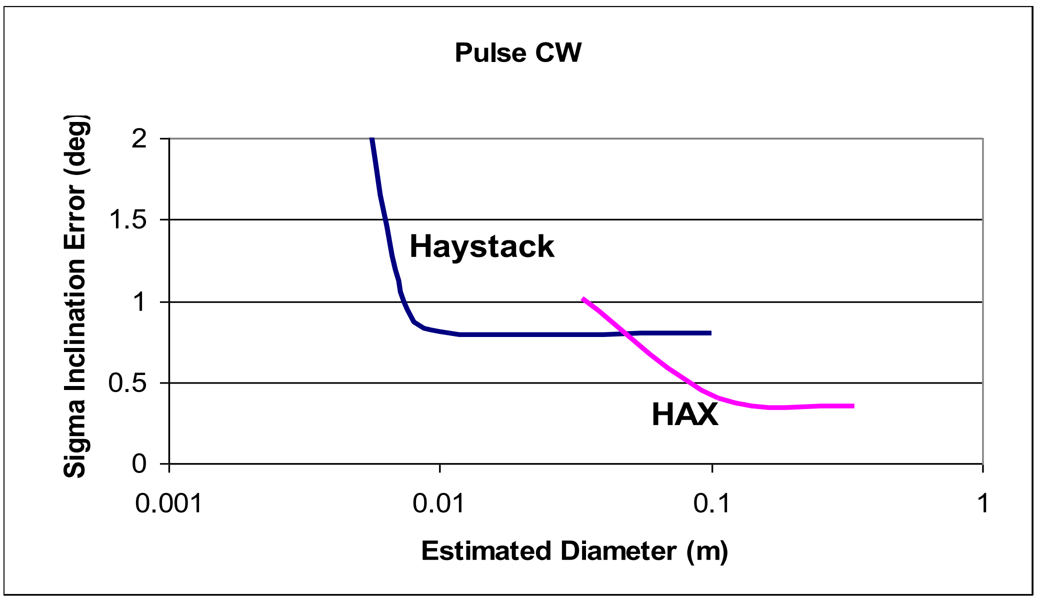

Figure 14. Haystack and HAX Inclination Error – 1000 km

Figure 14. As shown the Haystack inclination error is sufficient to provide an estimate of the inclination of the debris detections made in the 2003 debris campaign. Indeed, inclination of the debris detected was documented, as noted earlier.

The inclination error for Haystack and HAX was also computed using the parameters of the LFM tracking mode recommended for cued and/or stare and chase operation. In this case since range is now measured, the relationship in (3) was used. The results are shown in Figure 15. Because of their small beamwidths the angular error for both radars is small. The errors are small enough so that the first term in (4) is much less than the second term. Thus, the range error defines the inclination error. Since track time and range error is the same for both radars, they both achieve the same inclination error for target sizes larger than their minimum sensitivity. It would seem that the 8 pulse integration for HAX is not sufficient to provide the sensitivity desired in the 1 to 5 cm region. However, it should be noted that even at 1 cm the inclination error is still only 0.06 deg. This may still be adequate for catalog maintenance.

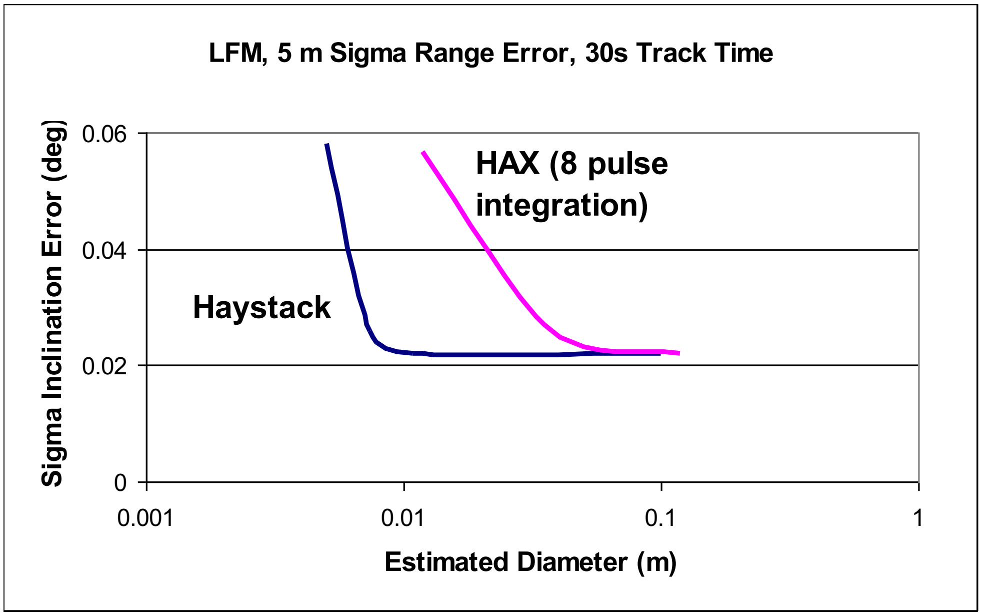

Figure 15. Haystack and HAX LFM Inclination Error - 1000 km

Small Debris Cataloging

To be able to avoid collision with 1 to 10 cm debris the element sets of the debris must be in the US Space Surveillance catalog. The catalog is maintained by a process of tasking radar to provide tracking data, processing the data, updating the debris element set and repeating the process. All tracks (observations) are correlated to the catalog. They either match and the update follows or they do not and are declared an Uncorrelated Target (UCT). Correlation with previously tracked UCTs is then made and the element set is either updated and the correlation process with the catalog continued or they are entered as another UCT[13].

The biggest challenge associated with tracks on small debris, is reacquisition on subsequent passes. Reacquisition must be attempted at the first available opportunity, specifically the first pass following the initial detection. Finding small debris after several orbits can be difficult due to atmospheric drag. Initial UCT element set accuracy should be accurate enough that other sensors in the network can acquire and track the debris. If other tracks are made the debris object may correlate with other (UCT) tracks of the debris and enter the catalog.

The criteria to determine track status is associated with the comparison of the estimated position of the debris object with those in the catalog. Correlation occurs if the object is within the association volume. The association volume used by the US Space

Command for associating tracks of known objects is a three dimensional box in the in-track, radial and out-of-plane position space centered on the predicted position. The nominal sides of this box are[14]:

In-track:	3 seconds (0.05 min)
Radial:	5 km
Out-of-Plane:	0.05 deg

If outside this volume the new UCTs (new debris tracks) will then be compared with other UCTs to determine if any UCTs correlate, as noted. The correlated UCTs can then be combined to develop a catalog entry sufficient for tasking updates from additional radars (if possible). Currently the criteria for UCT correlation can be 3 to 4 times that for catalog correlation (e.g., 0.2 min in-track, 0.2 deg. inclination).

It is envisioned that based on the increased density of space objects in the region from 1 to 10 cm the number of tracks (observations) and the processing required will increase dramatically. The criteria for a new uncorrelated target entry must be met to insure that false observations are not placed in the UCT file. Current criteria of the orbit parameters listed above were established primarily on the basis of detecting and tracking 10 cm to 1 meter objects (payloads and rocket bodies and large debris) and the nearest neighbor distance between these objects. If cataloging of debris objects in the 1 to 10 cm range is required, it might be necessary to establish unique criteria for this range. Criteria must be defined based on the density of the 1 to 10 cm population at the altitudes of interest. As an example, Figure 15 shows the debris flux calculated from the 2003 Haystack measurements[15]. The data illustrates two points. The first is that there is an order of

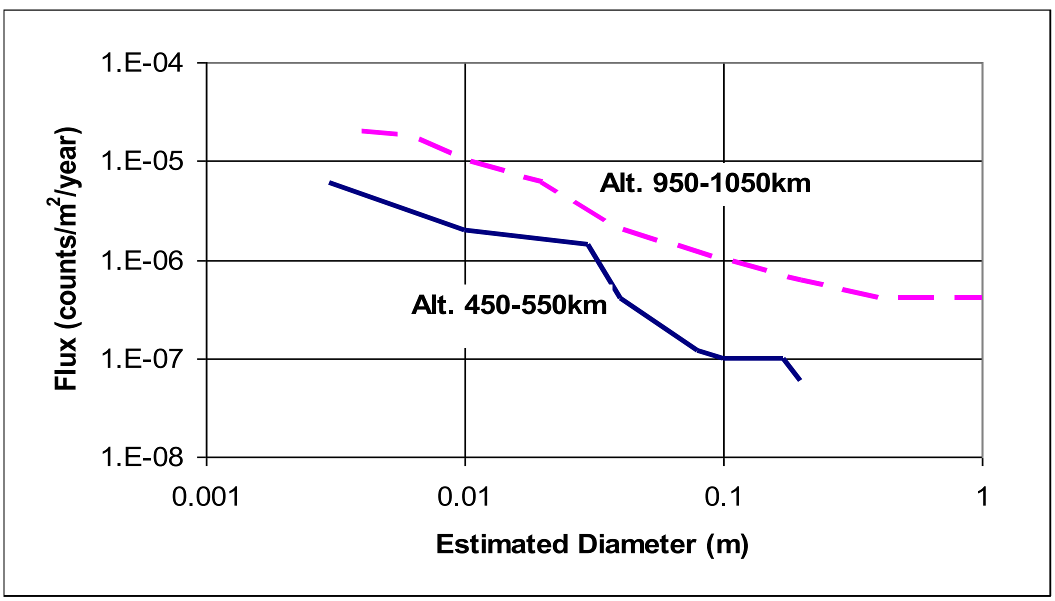

Figure 15. Haystack measured debris size distribution

magnitude increase in the density of objects from 10 cm to 1 cm at both the altitudes of interest shown. The second is that there is an order of magnitude density increase in going from low altitude (500 km) to the higher altitude (1000 km). Clearly the higher densities speak to establishing a UCT association volume smaller than currently being used.

Summary of Current Capability

Based on the brief analysis conducted here and the parameters estimated for the FPS-85 phased array, the radar has the capability to construct a high elevation "debris fence" to provide un-tasked detection and tracking of small debris. With the high fence the radar is able to provide measurements accurate enough to meet current UCT correlation criteria on 4 cm debris and larger to 1000 km. While this conclusion is based on using a LFM track waveform, it is for a single pulse. The sensitivity can be improved by a small degree by pulse integration to allow smaller debris to be tracked, but at the expense of multiple target detection and tracking. This option was not evaluated.

The HAX radar, depending on antenna slew rates, has the sensitivity to provide un-cued detection and tracking of small space debris in a possible stare and chase mode. With on-pulse modulation (LFM) and 30 sec track times accurate range as well as angle measurements can be provided to meet UCT correlation criteria, as presently defined for period (in-track) for 2 cm and larger debris to 1000km. And, with LFM and 30 sec tracks the Catalog correlation criteria as presently defined for inclination (out-of-plane) can be met for 2 cm and larger debris to 1000km. With additional track times it is possible to increase the period accuracy to meet the cataloging criteria.

With cuing the Haystack radar with LFM on-pulse modulation can update all small (1 to 10 cm) debris UCT element sets, with period and inclination prediction accuracy to meet all UCT correlation criteria over the altitude range of interest. With additional track time the stricter cataloging criteria can be met.

Observations/Recommendations

Future debris surveillance radar designed to catalog 1 to 10 cm debris for collision avoidance should operate in the S-band to C-band and have the sensitivity to detect/track 1 cm targets at 1800km. Ideally the radar should have agile beam capability to simultaneously search and track multiple debris targets to 1800 km with a minimum track time of 60 sec. A small debris catalog criteria must be established to correlate new debris UCTs for the high density (1 to 10 cm) altitude regimes. The Proposed US Air Force S-band Space Fence Concept might meet the debris tracking radar requirements and form the main element of a Space Debris Surveillance Network. The FPS-85 and HAX have the potential to contribute to a Space Debris Surveillance Network as secondary sensors. Haystack can provide RCS measurements and updates on established element set data when tasked.

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