Refinement of Parameters and Motion Forecast for High-Orbit Objects with a Big Area to Mass Ratio

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Introduction

The purpose of this report is to develop the methods and algorithms for determination of orbits of objects with a big area to mass ratio. Objects with a big area to mass ratio (AMR>1 m^2/kg) are usually located on high Earth orbits. Their orbits greatly evolve under the influence of solar radiation. The disturbing influence of solar radiation considerably exceeds disturbances caused by the eccentricity of the Earth gravity field and the influence of the Moon and Sun gravitation. Because of the lack of knowledge on the orientation and form of objects, the acceleration produced by the solar radiation pressure changes greatly in its value and direction. This makes it impossible to determine the orbit for long intervals of time and results in a considerable increase of the forecasting errors. A necessity arises to include in the number of the refined parameters the vector characterizing the solar radiation value and the direction of its pressure, along with the object state vector.

An approach is considered for the determination of the orbits and motion forecasting of objects with a big AMR. A limited span of the measurement interval in some cases is insufficient for the determination of a necessary set of parameters. The direction of the vector of acceleration caused by the solar radiation pressure can vary considerably, but in a limited range, deviate from the direction Sun-object. An algorithm is proposed for a joint determination of the object orbital parameters and the vector of additional acceleration, caused by the solar radiation pressure with an account of a priori information. An algorithm is described for formation of a weight matrix of a priori vector taking into account the range of its variation in value and direction. The questions are discussed of matching newly generated orbits of objects with a big AMR to the orbital catalog. A matching algorithm is proposed taking into account the errors in determination of objects' state vectors and the density of occupation of a considered part of orbital parameters space with other objects. An example is given of determination and forecasting of a real SO orbit and estimation of error forecasting on the basis of real measurements obtained by ground facilities.

1. Joint determination of a SO orbital parameters and parameters characterizing the influence of the solar radiation on the SO motion. General formulation of the problem

The set of refined parameters $\mathbf{Q}\{\mathbf{q},\mathbf{P}\}$ of motion of an object with a big AMR includes:

- six SO orbital elements $\mathbf{q}$
- components of the vector $\mathbf{P}$ of additional parameters characterizing the influence of the solar radiation pressure on the SO motion

The determination of the vector of refined parameters $\mathbf{Q}$ by the measurement data $\boldsymbol{\psi}_\mathbf{i}, i=1,2,...,N$ is carried out by a maximum-likelihood method by means of the functional minimization

$$\Phi(\mathbf{Q})=\sum_{i=1}^{N}\boldsymbol{\xi}_i^T(\mathbf{Q})\mathbf{W}_i\boldsymbol{\xi}_i(\mathbf{Q}) \tag{1}$$

where

$\xi_i(\mathbf{Q}) = \boldsymbol{\psi}_i - \boldsymbol{\psi}_i^c(\mathbf{Q}), i = 1, 2, ..., N$ are discrepancies of the measured and the calculated values of all types of measurements;

$\boldsymbol{\psi}_i$ — measurement;

$\boldsymbol{\psi}_i^c$ — its calculated analog;

$\mathbf{W}_i = \mathbf{K}_i^{-1}$ – measurement weight matrix;

$\mathbf{K}_i$ — covariance matrix of the measurement errors

Measurement $\boldsymbol{\psi}_i$ is considered as a vector, the components of which are, generally speaking, interconnected by a correlation dependence. Dimensions m_i of vectors $\boldsymbol{\psi}_i$ can differ. In a particular case, m_i can be equal to 1 (scalar measurement). It is supposed that vector measurements $\boldsymbol{\psi}_\mathbf{i}\left\{\psi_i^1, \psi_i^2, ..., \psi_i^m\right\}$ composing the functional (1) are not dependent on one another. At the same time, a presence of correlations is assumed between components ψ_i^j and ψ_i^k of vector $\boldsymbol{\psi}_i$. These correlations and the accuracy of measurement $\boldsymbol{\psi}_i$ are characterized by the covariance error matrix $\mathbf{K}_i$.

In this report the following is considered as possible measurement types:

- Distance
- Radial speed
- Angular measurements of right ascension and declination
- Priori data on the SO state vector
- Priori data on the orbital elements
- Priori data on the solar radiation pressure vector

An algorithm for the calculation of these measurements calculated values is specified in section 4.1. For minimization of functional Φ a standard iteration Newton method is used. At each k-th stage a system of equations is solved which is usually called a system of normal equations

$$\mathbf{A}\Delta\mathbf{Q} = \mathbf{b} \qquad (2)$$

where matrix A and vector $\mathbf{b}$ of the system of equations (2) are determined by the formulas

$$\mathbf{A} = \sum_{i=1}^{N}\left(\frac{\partial \xi_i}{\partial \mathbf{Q}}\right)^T \mathbf{W}_i \left(\frac{\partial \xi_i}{\partial \mathbf{Q}}\right)$$
$$\mathbf{b} = \sum_{i=1}^{N}\left(\frac{\partial \xi_i}{\partial \mathbf{Q}}\right)^T \mathbf{W}_i \xi_i \qquad (3)$$

and the parameters $\mathbf{Q}$ are refined:

$\mathbf{Q}_m = \mathbf{Q}_{m-1} + \Delta\mathbf{Q}$

The process of formation of a system of normal equations involves accumulation of the matrix, right parts and functional in proportion to the serial procession of incoming data. To make such accumulation possible it is necessary to have the following information for each successive measurement

- Dimension of vector of the measured parameter m (it is possible that $m = 1$). The dimension depends on the type of measurement and is determined after its retrieval.
- Measured vector $\boldsymbol{\psi}$ (scalar in a particular case)

- Weight matrix (weight in a particular case) characterizing the measurement error. The matrix is selected together with the measured parameter.
- Calculated vector $\boldsymbol{\psi}$ and a matrix of its derivatives $\frac{\partial \boldsymbol{\psi}}{\partial \mathbf{Q}}$ by the refined parameters vector

For each type of measurement a special module is developed providing calculation of the vector of its calculated value $\boldsymbol{\psi}^c$ and a matrix of derivates by the current vector of the SO kinematic parameters — $\frac{\partial \boldsymbol{\psi}^c}{\partial \mathbf{X}}$.

2. SO motion model and equations

SO motion model considers

- Earth gravitational influence represented by a sum of central and eccentric parts of the gravity field potential
- Moon and Sun gravitational influence
- Solar radiation pressure influence

In the inertial reference system EJ2000 connected with the equator and Earth equinox of epoch J2000 the SO motion equations are represented in the form

$$\dot{\mathbf{r}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = -\mu_E \frac{\mathbf{r}}{r^3} + A_{GCS}^{EJ2000} \mathbf{f}_{grav}\left(\mathbf{r}_{GCS}\right) + \mathbf{f}_{pm}\left(\mathbf{r}\right) + \mathbf{f}_{sp}\left(\mathbf{r}\right) \quad (4)$$

where

$\mathbf{r}$ — SV coordinates in the reference system EJ2000,

$\mathbf{r}_{GCS} = A_{EJ2000}^{GCS}\mathbf{r}$ — SV coordinates in the reference system GCS (Greenwich coordinate system),

$\mathbf{r}_{GCS}^{moon} = \mathbf{A}_{EJ2000}^{GCS}\mathbf{r}_{moon}$ — Moon coordinates in the reference system GCS,

$\mathbf{v}$ — SV speed vector in the reference system EJ2000,

$\mathbf{v}_{GCS} = A_{EJ2000}^{GCS}\mathbf{v}$ — SV speed vector in the reference system GCS,

μ_3 — Earth gravitation constant,

A_{EJ2000}^{GCS} — matrix of transfer from EJ2000 to GCS

$A_{GCS}^{EJ2000} = \left[A_{EJ2000}^{GCS}\right]^T$ — matrix of transfer from GCS to EJ2000 transposed in relation to matrix A_{EJ2000}^{GCS}

$\mathbf{f}_{gf}$ — (gravity field) function for calculation of the SV disturbing acceleration caused by eccentricity of the gravitation field

$\mathbf{f}_{pm}$ — (point mass) function for calculation of the SV disturbing acceleration caused by the influence of the gravitation fields of the Sun, Moon, and planets

$\mathbf{f}_{sp}$ — (solar pressure) function for calculation of the SV disturbing acceleration caused by the solar radiation pressure.

The first three sums in the right parts of the equations (4) are conditional on the influence by the Earth, Moon, and Sun gravitation fields on the SV motion. Formulas for calculation of these disturbances are well described in literature (for example [1]). A detailed description of a particular application of algorithms is beyond the scope of this report. The next section describes a model of the solar radiation influence on the SO motion.

3. Model of disturbing acceleration caused by the solar radiation influence

The model used for the calculation of the disturbing accelerations caused by the solar radiation pressure for objects with a big AMR is based on the following assumptions:

- The disturbing acceleration direction can in some limits differ from the direction Sun-SO.
- On different trajectory sections the disturbing acceleration value and direction can change discontinuously.
- The disturbing acceleration value and direction are described by a 3-dimension vector of parameters $\mathbf{p}_i$ which preserves a constant value within a section.
- On the selected interval of time the SO trajectory is continuous and described by a system of differential equations within one model and one SO state vector related to a specified moment of time.

In Figure 1, as an example, a section of the SO trajectory is schematically shown covering the interval of time $[t_4, t_0]$. The state vector $\mathbf{X}_0\{\mathbf{r}_0, \mathbf{v}_0\}$ determining the SO trajectory is related to the end of that interval t_0. The interval $[t_4, t_0]$ is divided into four sections $[t_4, t_3]$, $[t_3, t_2]$, $[t_2, t_1]$, $[t_1, t_0]$ ($t_4 < t_3 < t_2 < t_1 < t_0$). The parameters vector $\mathbf{p}_i$ determines the solar radiation pressure influence on the SO motion on the section $[t_i, t_{i-1}]$

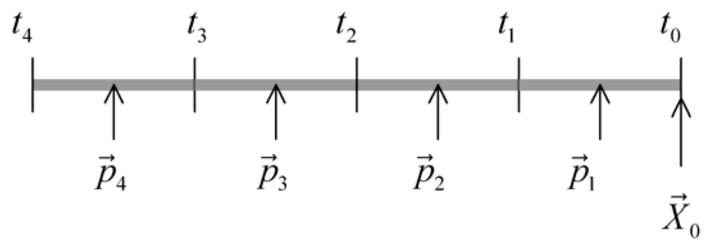

Figure 1. Diagram for making a model of the solar radiation pressure consideration

The force F of the solar radiation pressure at the distance 1AU≈150·10^9 m (Earth orbital radius), on a perfectly reflecting surface with the area of 1 m^2 located orthogonally to the direction towards Sun, amounts approximately to 9.1·10^{-6} N. For a perfectly absorbing surface this force is approximately equal to 4.5·10^{-6} N. In a general case F can be expressed by the formula [2]

$$F \approx c_r 4.5 \cdot 10^{-6}\ \mathrm{N}$$

where c_r — reflection index depending on the surface properties: $c_r = 1$ corresponds to complete radiation absorption, $c_r = 2$ corresponds a complete radiation reflection.

Let A be the area of the SO midsection in relation to the direction towards Sun, and m is its mass.

In the case when the surface completely absorbs the solar radiation flux, the disturbing acceleration $\mathbf{f}_{sp}^0$ is directed along the vector Sun-SO and its value amounts to $4.5 \cdot 10^{-6} \dfrac{A}{m}$ m/s^2. As for a near-Earth SO the direction Sun-SO is a little different from the direction Sun-Earth, the disturbing acceleration vector can be calculated by the formula

$$\mathbf{f}_{sp}^{0} = \frac{\mathbf{r}-\mathbf{r}_{sun}}{|\mathbf{r}-\mathbf{r}_{sun}|} 4.5\cdot 10^{-6}\frac{A}{m} \approx -\frac{\mathbf{r}_{sun}}{|\mathbf{r}_{sun}|} 4.5\cdot 10^{-6}\frac{A}{m} \quad (5)$$

where

$\mathbf{r}_{sun}$ — Sun geocentric position in the reference system EJ2000.

In the real situation the SO surface does not completely absorb sunlight. The direction of the disturbing acceleration vector $\mathbf{f}_{sp}$ can be different from the direction of the Sun radiation flux and its modulus can be more than the modulus of vector $\mathbf{f}_{sp}^{0}$. In this case the disturbing acceleration vector can be calculated by the formula:

$$\mathbf{f}_{sp} = 4.5\cdot 10^{-6}\left(p_e\mathbf{b}_e + p_n\mathbf{b}_n + p_s\mathbf{b}_s\right) = 4.5\cdot 10^{-6} B\mathbf{p} \quad (6)$$

where

$\mathbf{p} = \{p_e, p_n, p_s\}^T$ is a vector of parameters determining the solar radiation pressure influence

p_e is a component of this vector lying in the ecliptic plane and orthogonal to the direction towards Sun

p_n is a component of vector $\mathbf{p}$ directed orthogonally to the ecliptic plane

p_s — is a component of vector $\mathbf{p}$ directed from the Sun towards Earth

Matrix $B\begin{Bmatrix}\mathbf{b}_e \\ \mathbf{b}_n \\ \mathbf{b}_s\end{Bmatrix}$ is composed of vectors $\mathbf{b}_e$, $\mathbf{b}_n$, $\mathbf{b}_s$ which are calculated by the formulas:

$\mathbf{b}_s = -\frac{\mathbf{r}_{sun}}{|\mathbf{r}_{sun}|}$ is a block vector directed from Sun towards Earth

$\mathbf{b}_n = \frac{\mathbf{r}_{sun}\times\mathbf{v}_{sun}}{|\mathbf{r}_{sun}\times\mathbf{v}_{sun}|}$ is a block vector directed orthogonally to the ecliptic plane

$\mathbf{b}_e = \mathbf{b}_s \times \mathbf{b}_n$ is a block vector lying in the ecliptic plane and orthogonal to the direction towards the Sun.

Vectors $\mathbf{b}_e$, $\mathbf{b}_n$, $\mathbf{b}_s$ represent three mutually orthogonal directions. Projections of vector $\mathbf{p}$ on these directions with an accuracy of a multiplier, comply with the components of the disturbing acceleration $\mathbf{f}_{sp}$. If an object completely absorbs radiation, the components p_e and p_n, directed orthogonally to the light flux, are equal to zero, and the component p_s is equal to the area to mass ratio:

$$\mathbf{p} = \left\{0, 0, \frac{A}{m}\right\}$$

The actual values p_e, p_n, p_s are more or less different from these values.

4. Algorithm for the joint determination of a SO orbital parameters and parameters characterizing the influence of the solar radiation on the SO motion

In section 1 there is a summary description of a standard procedure for the refinement of a vector of the estimated parameters $\mathbf{Q}$ by the measurement data. Formulas (2) and (3) provide the minimization of the functional (1) by the iteration method. Although, for the solution of a particular problem it is necessary to develop an algorithm for the calculation of the measurement calculated values $\boldsymbol{\psi}^c$, $i = 1,2,...,N$ by the specified vector of the refined parameters $\mathbf{Q}$, and an algorithm for the calculation of a matrix of partial derivatives $\dfrac{\partial \boldsymbol{\psi}_i^c}{\partial \mathbf{Q}}$ from the calculated parameters by the refined parameters $\mathbf{Q}$. These algorithms are described below.

4.1. Calculation of a measured parameter of a matrix of its partial derivatives by the specified vector of the estimated parameters

A procession interval $[t_m, t_0]$ is considered divided into m subintervals by the moments of time $t_0 > t_1 > ... > t_m$. The total of the refined parameters is defined by the vector $\mathbf{Q}\{\mathbf{q}_0, \mathbf{p}_1, ..., \mathbf{p}_m\}$, where $\mathbf{q}_0$ is the SO orbital elements attributed to the moment of time t_0, $\mathbf{p}_i$ are the parameters determining the light pressure influence in the interval $[t_i, t_{i-1}]$.

In the moments of time $t_1', t_2', ..., t_N'$ the measurements $\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, ..., \boldsymbol{\psi}_N$ are carried out.

In the moment of time t_k' located within the interval $[t_i, t_{i-1}]$ the dependence of the calculated value $\boldsymbol{\psi}_k^c$ on the set of the refined parameters $\mathbf{Q}$ is determined by the formula

$$\boldsymbol{\psi}^c\left(\mathbf{X}(t_k', \mathbf{q}_0, \mathbf{p}_i, \mathbf{p}_{i-1}, ..., \mathbf{p}_1)\right) = \boldsymbol{\psi}^c\left(\mathbf{X}\left(t_k', \mathbf{p}_i, \vec{X}\left(t_{i-1}, \mathbf{p}_{i-1}, \mathbf{X}\left(...\mathbf{X}\left(t_1, \mathbf{p}_1, \mathbf{X}_0(\mathbf{q}_0)\right)...\right)\right)\right)\right) \quad (7)$$

From this representation it is obvious that the calculated value $\boldsymbol{\psi}^c$ depends on the orbital elements $\mathbf{q}_0$, and the set of parameters $\mathbf{p}_i, \mathbf{p}_{i-1}, ..., \mathbf{p}_1$ characterizing the solar radiation pressure in all intervals after the moment t_k'.

Expressions of the type $\mathbf{X}(t, \mathbf{p}_j, \mathbf{X}_{j-1})$ denote the dependence of the SO state vector $\mathbf{X}$, attributed to the moment of time t on the value of the vector $\mathbf{p}_j$ in the interval $[t_j, t_{j-1}]$ and the state vector $\mathbf{X}_{j-1}$ attributed to this interval beginning time — t_{j-1}. The calculated realization of this dependence is carried out by a numerical integration of the motion equations (4) with the constant value of the vector $\mathbf{p}_j$.

The representation $\mathbf{X}_0(\mathbf{q}_0)$ expresses the dependence of the SO state vector in the moment of time t_0 on the orbital elements $\mathbf{q}_0$. This vector is calculated by the formulas of Kepler's undisturbed motion.

Partial derivatives from the measured functions $\boldsymbol{\psi}^c$ by the estimated parameters are calculated by the formulas:

$$\frac{\partial \boldsymbol{\psi}^c}{\partial \mathbf{Q}} = \frac{\partial \boldsymbol{\psi}^c}{\partial \mathbf{X}} \frac{\partial \mathbf{X}}{\partial \mathbf{Q}}$$

4.2. Calculation of derivatives from the SO current state vector by the vector of estimated parameters

The matrix of derivatives $\frac{\partial \mathbf{X}}{\partial \mathbf{Q}}$ from the SO state vector $\mathbf{X}$ in the current moment of time t by the specified vector of the estimated parameters $\mathbf{Q}$ consists of several blocks as shown in Figure 2. If $\boldsymbol{\psi}$ is changed in the moment of time t in k-th trajectory subinterval, then

$$\frac{\partial \mathbf{X}}{\partial \mathbf{p}_{k+1}} = 0, \ldots, \frac{\partial \mathbf{X}}{\partial \mathbf{p}_m} = 0 \qquad t_{k+1} < t \le t_k \tag{8}$$

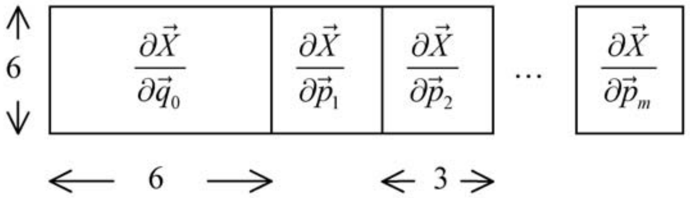

Figure 2. Matrix of derivatives $\frac{\partial \mathbf{X}}{\partial \mathbf{Q}}$

Table 1 gives an idea of the derivatives location in the matrix blocks in various subintervals of the SO trajectory. The subintervals are denoted by the symbol τ with the index: τ_k corresponding to the subinterval $[t_k, t_{k-1}]$. To form various types of derivatives, three different calculation methods are used. The table cells are marked in different ways depending on the calculation method used for calculation of derivatives:

- Off-diagonal matrix blocks ($k > i$) are equal to zero, because the SO state vector does not depend on the light pressure influence on the previous motion sections.
- The diagonal table cells are surrounded by the black frames. Their feature is that the number of a section k, on which the derivatives are calculated, is equal to the number i of the refined vector $\mathbf{p}_i$ ($k = i$). These derivatives are calculated in the process of the integration of the equations in variations with the renewal of the initial conditions matrix at each transfer to a new subinterval.
- The double frame marks the derivatives $\frac{\partial \mathbf{X}}{\partial \mathbf{q}_0}$ from the vector of the current SO kinematic parameters $\mathbf{X}$ by the refined elements of its orbit $\mathbf{q}_0$. They are calculated by continuous numerical integration of the equations in variations, without renewal of the initial conditions.
- In the grey background the formulas are printed in the table cells under the diagonal — $k < i$. In this calculation variant the derivatives are calculated by multiplication of the three matrixes

$$\frac{\partial \mathbf{X}}{\partial \mathbf{p}_i} = \frac{\partial \mathbf{X}}{\partial \mathbf{X}_{k-1}} \frac{\partial \mathbf{X}_{k-1}}{\partial \mathbf{p}_i} = \frac{\partial \mathbf{X}}{\partial \mathbf{q}_0} \frac{\partial \mathbf{q}_0}{\partial \mathbf{X}_{k-1}} \frac{\partial \mathbf{X}_{k-1}}{\partial \mathbf{p}_i} = \frac{\partial \mathbf{X}}{\partial \mathbf{q}_0} \left(\frac{\partial \mathbf{X}_{k-1}}{\partial \mathbf{q}_0} \right)^{-1} \frac{\partial \mathbf{X}_{k-1}}{\partial \mathbf{p}_i} \tag{9}$$

his case the derivatives by the parameters $\mathbf{p}_j$ are used, as obtained in the previous trajectory section.

Table 1

Estimated parameters						
$\mathbf{p}_m$	…	$\mathbf{p}_2$	$\mathbf{p}_1$	$\mathbf{q}_0$		
	…	0	$\frac{\partial \mathbf{X}}{\partial \mathbf{p}_1}$	$\frac{\partial \mathbf{X}}{\partial \mathbf{q}_0}$	τ_1	Subintervals
0	…	$\frac{\partial \mathbf{X}}{\partial \mathbf{p}_2}$	$\frac{\partial \mathbf{X}}{\partial \mathbf{p}_1}$	$\frac{\partial \mathbf{X}}{\partial \mathbf{q}_0}$	τ_2	
…	…	…	…	…	…	
$\frac{\partial \mathbf{X}}{\partial \mathbf{p}_m}$	…	$\frac{\partial \mathbf{X}}{\partial \mathbf{p}_2}$	$\frac{\partial \mathbf{X}}{\partial \mathbf{p}_1}$	$\frac{\partial \mathbf{X}}{\partial \mathbf{q}_0}$	τ_m	

5. Software implementation of the algorithm for joint determination of a SO orbital parameters and parameters characterizing the solar radiation influence on the SO motion

From the proposed methods the algorithm for the joint determination of a SO orbital parameters and the parameters characterizing the solar radiation influence on the SO motion is implemented in software. For the algorithm applicability verification, a SO is selected, discovered in summer 2008 (number 90121 in the catalog of the Institute of Applied Mathematics of Russian Academy of Sciences). Its tracking was started, but several times it was lost and found again. To demonstrate the algorithm work, an interval from 2009/06/17 to 2009/07/30 is selected. For estimation of the method effectiveness, the calculations were carried out in two variants:

1. By the traditional method: refinement of 7 parameters — 6 components of the SO state vector and one value of the solar radiation pressure ratio with the assumption that it's directed on the line Sun-SO;
2. Refinement of 9 parameters under the developed method:6 components of the SO state vector and 3 components of the vector characterizing the solar radiation influence on the SO motion in the following directions:
   - Along the line Sun-SO,
   - Orthogonal to the line Sun-SV, in the ecliptic plane,
   - Orthogonal to the line Sun-SV, orthogonal to the ecliptic plane.

In processing measurements there is always a problem of a measurement interval selection. A short measurement interval does not provide sufficient accuracy of the refined parameters, characterized by the a priori covariance error matrix. A long measurement interval leads to the occurrence of discrepancies due to the nonconformity of the accepted model to the real SO motion. As a criterion of the processing quality, the following value is taken

$$q=\sigma_{o-c}\sqrt{\frac{\partial a}{\partial \mathbf{X}_0}\mathbf{C}_{\mathbf{x}}\left(\frac{\partial a}{\partial \mathbf{X}_0}\right)^T}$$

where

$\mathbf{C_x}$ is the a priori covariance error matrix

a is the major orbital semi-axis

$\mathbf{X}_0$ is the SO state vector attributed to the moment of refinement t_0

σ_{o-c} is the root-mean square weighted value of the deviation of the calculations from their calculated analogs.

The value q gives an idea of the obtained solution quality: it decreases with a decrease of the a priori semi-axis error and increases with an increase of the calculation discrepancies, if the accepted model ceases to conform to the SO real motion.

The comparison of the effectiveness of the SO motion parameters determination, using parametric problems 7 and 9, was carried out by the following pattern (Figure 3). The last day of the procession interval was excluded from the procession and was used for the estimation of the solution quality as a control interval. The smaller value of the root-mean square discrepancies in the control interval is represented by a better solution, from the point of calculation of the target destination for the following night. The refined state vector was attributed to the moment of the last measurement included in procession, i.e. one day before the end of the measurement interval. The parameters were refined by the increasing measurement interval by means of engaging earlier measurements in procession. At each such interval 7 and 9 parameters were refined. The obtained data are specified in Tables 2 and 3. For comparison of the results of 7 and 9-parametric problems, the data of these tables are combined and represented in Table 4.

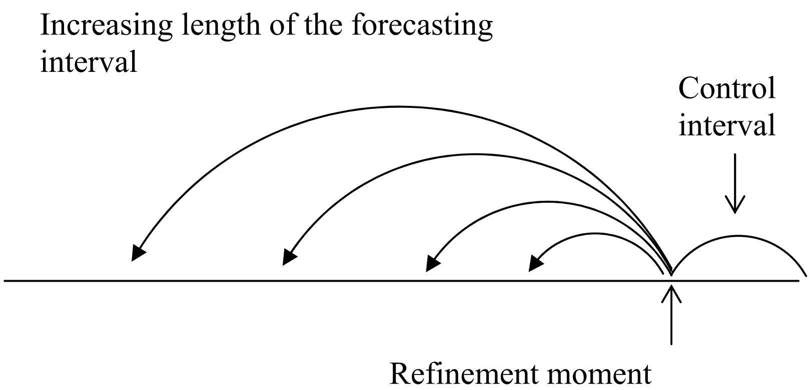

Figure 3

Table 2. Refinement of 7 parameters

Year 2009	Area to mass ratio	Mean weighted discrepancies	q-quality criterion	Average measurement deviations in the last interval (seconds of arc)
07/28-07/29	28.1508	0.5366	0.000647	314.3
07/27-07/29	28.34854	0.9103	0.000564	500.5
07/25-07/29	26.07779	2.2140	0.000145	253.9
07/24-07/29	25.44437	9.0488	0.000436	106.1

07/22-07/29	24.98914	9.0545	0.000146	51.4
07/21-07/29	24.96849	8.4534	0.000135	51.7
07/17-07/29	25.99957	10.2770	0.000083	217.0
07/16-07/29	25.83116	10.7606	0.000081	191.1
07/14-07/29	26.11147	**10.2065**	**0.000028**	**161.3**
7/13-07/29	26.07779	10.7191	0.000030	167.5
07/11-07/29	26.18535	16.2660	0.000037	247.3
07/10-07/29	26.12234	16.1309	0.000033	251.3
07/09-07/29	26.10387	16.2201	0.000032	243.9
07/08-07/29	26.12017	15.8457	0.000030	244.1
07/07-07/29	26.12668	15.8964	0.000029	251.0
07/05-07/29	26.24946	20.6431	0.000033	324.7

Table 3. Refinement of 9 parameters

	Area to mass ratio					
Year 2009	In the direction Sun-SO	In the ecliptic plane	Orthogonally to the ecliptic plane	MSD	q	Average deviations
07/28-07/29	17.92264	-14.1308	-6.5819	0.5252	0.019023	716.2
07/27-07/29	12.1545	-23.4844	-10.8963	0.7215	0.006577	1108.4
07/25-07/29	23.16927	-9.53716	-4.6414	0.7738	0.000557	110.6
07/24-07/29	30.68774	10.34116	3.5419	6.4424	0.002450	546.4
07/22-07/29	25.70187	1.511299	0.1586	8.6071	0.000586	112.1
07/21-07/29	25.54107	0.846371	-0.1575	8.1666	0.000338	98.2
07/17-07/29	25.04781	0.362886	-0.1401	8.1893	0.000108	62.0
07/16-07/29	25.2488	0.206432	-0.08148	10.4331	0.000122	72.9
07/14-07/29	**26.0854**	**0.001086**	**0.02933**	**10.2102**	**0.000050**	**145.1**
07/13-07/29	26.04628	-0.00217	0.07388	10.6226	0.000051	133.7
07/11-07/29	26.05606	0.011951	0.2542	14.9034	0.000068	103.1
07/10-07/29	26.06475	-0.02064	0.3139	14.5065	0.000057	112.7
07/09-07/29	26.05932	-0.04563	0.3270	14.9430	0.000059	122.2
07/08-07/29	26.11256	-0.06628	0.3346	14.7600	0.000054	137.7
07/07-07/29	26.11039	-0.04998	0.3205	14.6828	0.000053	131.5
07/05-07/29	26.17992	0.05867	0.2009	20.0063	0.000072	136.7

Table 4. Comparison of results of 7 and 9-parametric problems

Procession interval length	Year 2009	Average deviations in the control section	Average deviations in the control section
-28	07/28-07/29	314.3	716.2
-27	07/27-07/29	500.5	1108.4
-25	07/25-07/29	253.9	110.6

-24	07/24-07/29	106.1	546.4
-22	07/22-07/29	51.4	112.1
-21	07/21-07/29	51.7	98.2
-17	07/17-07/29	217.0	62.0
-16	07/16-07/29	191.1	72.9
-14	07/14-07/29	**161.3**	**145.1**
-13	07/13-07/29	167.5	133.7
-11	07/11-07/29	247.3	103.1
-10	07/10-07/29	251.3	112.7
-9	07/09-07/29	243.9	122.2
-8	07/08-07/29	244.1	137.7
-7	07/07-07/29	251.0	131.5
-5	07/05-07/29	324.7	136.7

In Figure 4 these deviations are represented in the graphical form. The crimson line corresponds to the solutions of the 7-parametric problem (one parameter for description of the solar radiation influence). The blue line corresponds to the solutions of the 9-parametric problem (3 additional parameters). It is obvious that the forecasting error in the second variant is better as the measurement interval length is more than 10 days.

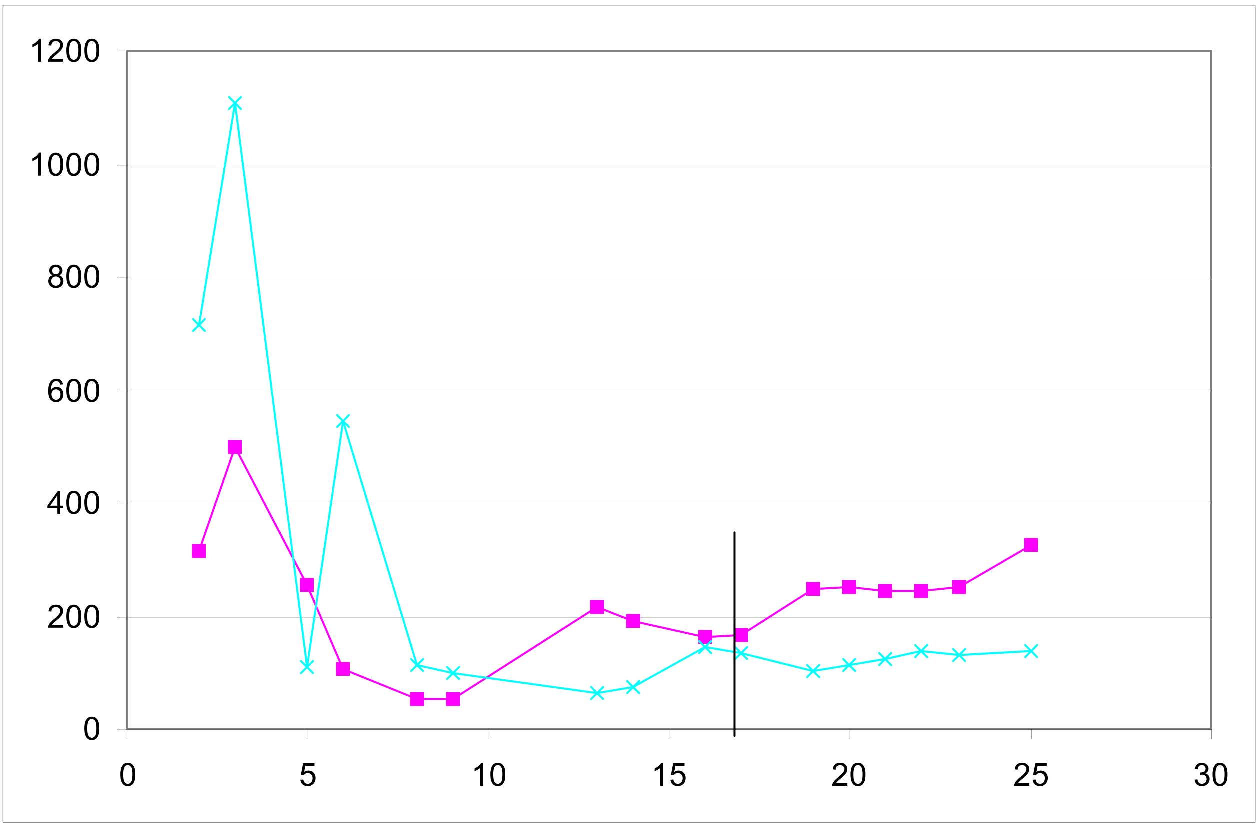

Figure 4. The axis of abscissas — time in days from the beginning of the measurement interval. The axis of ordinates — average deviations of the measured values from the calculated ones (seconds of angle). The vertical line marks the optimal interval length.

References

1. David A. Vallado, 2007, Fundamentals of Astrodynamics and Applications
2. Wertz, J.R. and W. J. Larson, 1999, Introduction to Astrodynamics, p.145