Uncertainty in Probability of Collision Calculations

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ABSTRACT

This paper investigates the magnitude and impact of covariance estimates with respect to collision avoidance predictions. The utility and effectiveness of collision avoidance predictions is directly dependent on the quality of the underlying covariance assessments of each object's state. As a statistical measure, the covariance can vary greatly as a function of the number and type of observations, their precision, biases, modeling errors resident in the orbit determination (OD) process along with artifacts from the selected trajectory reconstruction method. Further complicating the ability to accurately predict close approaches is the fact that many objects do not have an associated covariance matrix provided with their published state. An examination of the sensitivity of covariance to the characteristics of the observations and processing technique is detailed along with determining the utility of collision assessments, which incorporate combinations of no covariance data, static uncertainty estimates, and fully qualified error ellipsoids.

Acronym List

EGM	Earth Gravity Model
MSIS	Mass Spectrometer and Independent Scatterer
NRL	Navy Research Laboratory
OD	Orbit Determination
RSO	Resident Space Object
RSS	Root Sum Square

1.0 INTRODUCTION

Collision prediction is dependent upon three covariance based processes: Covariance generation, Covariance propagation, and covariance assessment, including probability determination and miss distance calculations. Numerous techniques have been developed to address the propagation and assessment processes, while detailing the covariance generation process has been neglected, despite dominating the accuracy and confidence of any resulting calculation [1]. With little insight into the covariance generation process, the resulting published uncertainties can result in wildly varying solutions, which directly impact planning, operations, and resources as shown in Figure 1. Similarly, none, or an unknown knowledge of an object's covariance, can result in extremely conservative and potentially pathological predictions, which again can significantly limit or restrict operations [2]. Conversely, optimistic covariance data can dramatically increase the probability of collision [3]. This paper provides the details regarding the sources, dependencies, sensitivities, and overall statistical structure of an object's resulting covariance, and how additional information can be supplied to accurately assess the probability of collision between two objects.

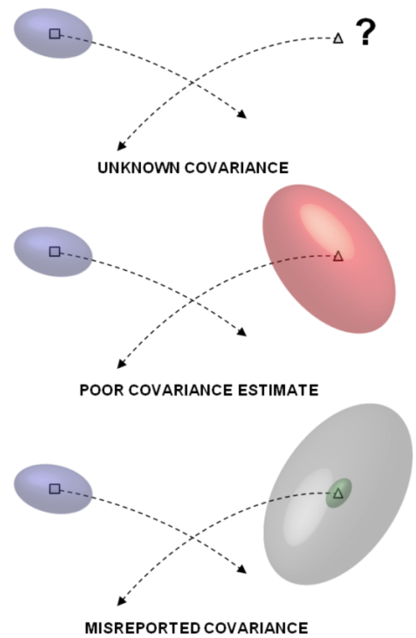

Figure 1: Covariance reporting varies

Through high fidelity modeling and sensitivity analysis, the components of the covariance generation process will be examined to include sensor and measurement characteristics, modeling uncertainties, and systematic errors and biases.

2.0 COVARIANCE REVIEW

As a statistical measure, covariance is directly affected by and reflects the metrics of the observations used to generate the solution. These observational metrics encompass measurement type (e.g. passive or active), quantity of observations, the quality of each observation and the overall set and the geometry between the observer and the target. Covariance solutions based solely on these observation based metrics result in optimistic or best case values for the uncertainty, since the solution is comprised completely of measurement noise.

To gain a realistic measure of the covariance, systematic errors, biases, model uncertainties and overall system complexity must be considered via a second covariance matrix to augment the observation, or solved for only solution. Many simulations and operational processes neglect this second covariance, or at best set values within it incorrectly. The source for the consider covariance matrix comes from two sources – published uncertainties for certain models and parameters (e.g. MSIS-00 drag model or the dominant J2 zonal harmonic from EGM-96) or from calibration of the system.

By correctly accounting for and combining the solved for (observational) and consider (systematic) covariances properly, a more realistic representation of the object's uncertainty can be generated. As illustrated in Figure 2 the resulting combined covariance can have a dramatically different orientation and magnitude from the solved for only covariance.

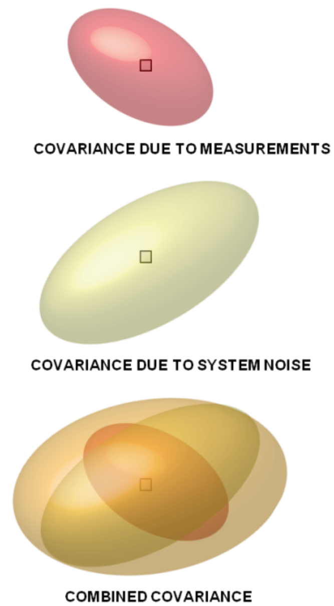

Figure 2: Covariance Components

3.0 COVARIANCE CONSIDERATIONS

As better observations are included in the solution (i.e. active measurements, increased quantity and quality and improved geometry), the solved for only covariance solution improves and the resulting uncertainty decreases. An often erroneous assumption made regarding solved for only solutions, is the idea that more observations is always better and that by including additional measurements the uncertainty can be reduced even further. Though the solved for only solution continues to decrease with each new measurement, the resulting consider matrix is at best maintaining its shape and orientation, or worst changing in size and direction rapidly to accommodate the increase in complexity in the system. Even in a single sensor system there are critical systematic errors and biases that must be accounted for such as sensor location errors, biases, timing errors, model errors and uncertainties [4]. Each of these elements contributes to the consider matrix and precludes the combined covariance matrix from approaching zero uncertainty for data or sensor rich configurations. Figure 3 shows this increase in covariance due to complexity in detail.

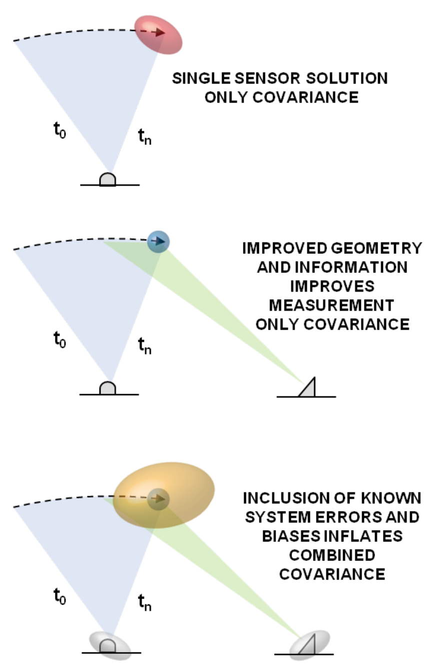

Figure 3: Covariance and Complexity

4.0 COVARIANCE TEST CASE

To illustrate the impact of not including consider terms in the reported covariance of an object, a single sensor tracking a single target case was generated. This case consists of either a telescope or radar tracking a large satellite placed into the wrong orbit. The failed launch case demonstrates the utility of a single sensor vs. a large tracking network and precludes the use of a priori or historical data to shape the solution. In this case a single satellite from a failed launch overflies a single site from horizon to horizon. To address sensitivities in the solution due to geometry, along with number, quality and type of observations (angles only or angles and range), the target's altitude was varied from 400 to 1000km. The consider matrix was generated using site uncertainty parameters, timing errors, model uncertainties per published reports, sensor biases and drag uncertainty of 3% per NRL MSIS-00. Table 1 and Figure 4 details the set and processing of the data. The resulting products included all three covariance matrices: solved for only, consider, and combined solved for plus consider. Each covariance was recorded in terms of its individual components (radial, in-track, cross-track) and the RSS of the three values for an overall spherical summary of the uncertainty.

Table 1: Test Case Setup

- Observation metrics:
  - Type: Active (radar: azimuth, elevation and range) vs. Passive (telescope: angles only)
  - Rate: 1 point/s, 10 points/minute, 5 points/minute
  - Quality (1 sigma):
    - *Active: 0.01 degrees for angles and 10m for range*
    - *Passive: 10 arc-seconds*
  - Geometry: Vary over flight duration from horizon maximum elevations spanning 5-85 degrees
- Consider Parameters:
  - 1 m site uncertainty
  - 1 micro second timing error
  - model uncertainties as reported by EGM-96
  - Sensor bias uncertainties at 10% 1 sigma values
  - 3% drag uncertainty per NRL MSIS-00

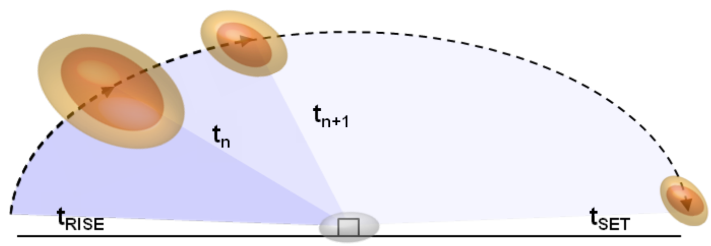

Figure 4: Failed Launch Overflight

Over the 600km range of altitudes, Figure 5 shows how significantly geometry impacts the number of observations as a function of elevation. Of interest in the chart is that regardless whether it is a low or high altitude vehicle, once the target is 45 degrees above the horizon the incremental information gained for each degree toward zenith is minimal. This trend is shown in Tables 2 and 3. Both tables show the uncertainty due to a single dish radar at the end of the overflight (at set) and propagated one orbit later to allow another sensor to acquire and track it. The difference between 2 and 3 is that 2 details the solved for only and

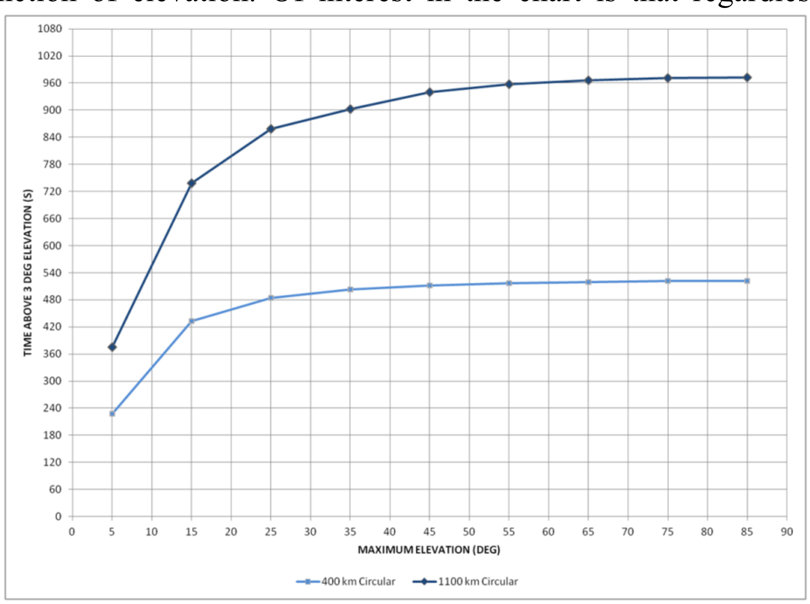

Figure 5: LEO Overflight Times

3 provides the combined covariance including the consider terms from Table 1.

Table 2: Active Sensor Solved for Only Covariance

END OF PASS RSS 1 SIGMA COVARIANCE (KM)

OPTIMISTIC SOLUTION END OF OBS

ALTITUDE (KM)	5	15	25	35	45	55	65	75	85
1000	0.066	0.034	0.027	0.025	0.023	0.022	0.022	0.019	0.012
900	0.064	0.033	0.027	0.024	0.023	0.022	0.021	0.018	0.012
800	0.062	0.032	0.026	0.024	0.022	0.022	0.021	0.017	0.012
700	0.059	0.031	0.025	0.023	0.022	0.021	0.020	0.016	0.011
600	0.056	0.030	0.024	0.022	0.021	0.020	0.019	0.015	0.011
500	0.053	0.029	0.024	0.022	0.021	0.020	0.017	0.014	0.010
400	0.049	0.027	0.023	0.021	0.020	0.018	0.015	0.013	0.010

MAX ELEVATION (DEG)

1 REV PREDICT RSS 1 SIGMA COVARIANCE (KM)

OPTIMISTIC SOLUTION 1 REV PREDICT

ALTITUDE (KM)	5	15	25	35	45	55	65	75	85
1000	0.720	0.349	0.268	0.213	0.168	0.128	0.093	0.067	0.056
900	0.736	0.347	0.264	0.210	0.165	0.126	0.092	0.067	0.058
800	0.748	0.344	0.260	0.206	0.162	0.124	0.092	0.068	0.059
700	0.764	0.339	0.254	0.201	0.158	0.122	0.091	0.070	0.062
600	0.783	0.331	0.247	0.194	0.154	0.120	0.091	0.072	0.065
500	0.800	0.321	0.237	0.187	0.149	0.117	0.092	0.075	0.069
400	0.822	0.307	0.226	0.179	0.144	0.116	0.094	0.080	0.076

MAX ELEVATION (DEG)

Table 3: Active Sensor Combined Covariance

REALISTIC SOLUTION END OF OBS

ALTITUDE (KM)	5	15	25	35	45	55	65	75	85
1000	0.404	0.303	0.233	0.185	0.150	0.120	0.087	0.056	0.036
900	0.379	0.280	0.214	0.169	0.137	0.109	0.079	0.051	0.034
800	0.354	0.257	0.194	0.152	0.123	0.098	0.072	0.047	0.032
700	0.326	0.233	0.174	0.135	0.109	0.087	0.064	0.043	0.030
600	0.298	0.208	0.153	0.118	0.095	0.076	0.056	0.039	0.028
500	0.267	0.181	0.131	0.101	0.081	0.065	0.049	0.035	0.026
400	0.233	0.152	0.109	0.083	0.066	0.053	0.041	0.031	0.025

MAX ELEVATION (DEG)

REALISTIC SOLUTION 1 REV PREDICT

ALTITUDE (KM)	5	15	25	35	45	55	65	75	85
1000	5.497	4.279	3.258	2.433	1.746	1.213	0.931	0.835	0.810
900	5.351	4.095	3.082	2.286	1.637	1.155	0.908	0.822	0.798
800	5.185	3.891	2.892	2.130	1.526	1.096	0.883	0.805	0.784
700	4.979	3.655	2.682	1.964	1.413	1.040	0.859	0.789	0.770
600	4.727	3.384	2.450	1.786	1.298	0.986	0.837	0.774	0.758
500	4.417	3.071	2.195	1.600	1.186	0.937	0.816	0.760	0.748
400	4.032	2.709	1.916	1.409	1.081	0.895	0.796	0.751	0.743

MAX ELEVATION (DEG)

The distinct color change between the two tables clearly illustrates the 2-10x increase in uncertainty due to the consider matrix being combined with the solved for only solution. Tables 4 and 5, illustrate the same distinct results, but through passive observations collected from a telescope.

Table 4: Passive Sensor Solved for Only Covariance

END OF PASS RSS 1 SIGMA COVARIANCE (KM)

OPTIMISTIC SOLUTION END OF OBS

ALTITUDE (KM)	5	15	25	35	45	55	65	75	85
1000	0.423	0.074	0.050	0.042	0.039	0.038	0.037	0.037	0.037
900	0.429	0.074	0.050	0.043	0.040	0.038	0.038	0.038	0.038
800	0.430	0.075	0.051	0.044	0.041	0.039	0.039	0.039	0.039
700	0.434	0.076	0.052	0.045	0.042	0.040	0.040	0.040	0.040
600	0.442	0.077	0.053	0.046	0.043	0.042	0.041	0.041	0.041
500	0.438	0.078	0.054	0.047	0.044	0.043	0.043	0.043	0.043
400	0.434	0.079	0.056	0.049	0.047	0.045	0.045	0.045	0.045

MAX ELEVATION (DEG)

1 REV PREDICT RSS 1 SIGMA COVARIANCE (KM)

OPTIMISTIC SOLUTION 1 REV PREDICT

ALTITUDE (KM)	5	15	25	35	45	55	65	75	85
1000	20.678	3.315	2.143	1.791	1.641	1.573	1.546	1.549	1.563
900	21.852	3.487	2.280	1.910	1.757	1.686	1.667	1.664	1.679
800	22.752	3.697	2.441	2.055	1.896	1.826	1.799	1.804	1.818
700	23.962	3.965	2.630	2.240	2.073	2.003	1.977	1.983	1.998
600	26.614	4.272	2.885	2.470	2.299	2.228	2.200	2.205	2.222
500	27.962	4.716	3.228	2.793	2.608	2.536	2.509	2.517	2.531
400	30.142	5.298	3.729	3.265	3.083	2.996	2.977	2.962	2.983

MAX ELEVATION (DEG)

Table 5: Passive Sensor Combined Covariance

REALISTIC SOLUTION END OF OBS

ALTITUDE (KM)	5	15	25	35	45	55	65	75	85
1000	0.426	0.091	0.075	0.071	0.069	0.065	0.061	0.059	0.058
900	0.432	0.092	0.076	0.072	0.070	0.066	0.062	0.060	0.059
800	0.432	0.092	0.077	0.074	0.071	0.068	0.064	0.061	0.061
700	0.436	0.093	0.078	0.075	0.074	0.071	0.067	0.064	0.063
600	0.444	0.095	0.080	0.078	0.076	0.074	0.070	0.067	0.066
500	0.441	0.097	0.083	0.082	0.080	0.078	0.074	0.071	0.071
400	0.436	0.099	0.087	0.086	0.085	0.083	0.080	0.077	0.076

MAX ELEVATION (DEG)

REALISTIC SOLUTION 1 REV PREDICT

ALTITUDE (KM)	5	15	25	35	45	55	65	75	85
1000	20.740	3.800	2.947	2.732	2.577	2.397	2.199	2.041	1.995
900	21.919	4.030	3.181	2.975	2.829	2.650	2.459	2.290	2.241
800	22.827	4.313	3.464	3.269	3.131	2.959	2.761	2.594	2.543
700	24.047	4.674	3.808	3.642	3.512	3.347	3.153	2.980	2.930
600	26.716	5.102	4.269	4.119	3.999	3.843	3.645	3.471	3.417
500	28.084	5.723	4.886	4.766	4.661	4.511	4.320	4.144	4.084
400	30.294	6.566	5.782	5.697	5.630	5.475	5.289	5.093	5.034

MAX ELEVATION (DEG)

5.0 COLLISION GEOMETRY & PROBABILITY CALCULATIONS

In regions where the modeling uncertainty can be significant Figure 6, illustrates the resulting wildly varying covariance solutions due to measurement only, combined covariance and combined covariance, plus a more realistic error inherent in attempting to model drag at such a low altitude. Figure 6 also highlights the smoothing impacting of the consider matrix in flattening the curve once the systematic errors begin to dominate the solution. Table 6 lists the final values from Figure 6 and Table 6 displays the corresponding covariance values.

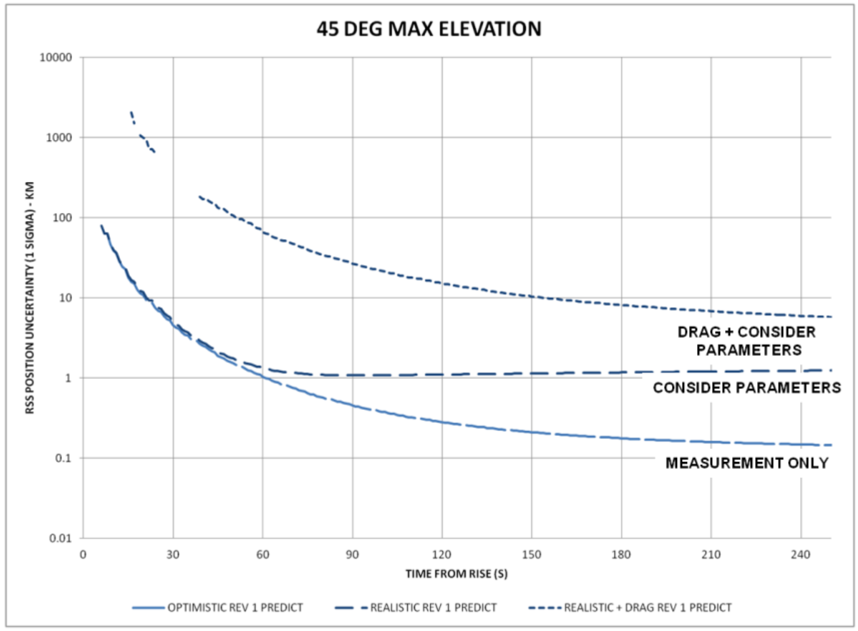

Figure 7: Types of Covariance

As mentioned above, multiple authors have done extensive investigations into the interaction between two objects. K. Chan and T. Alfriend provide simplifying assumptions to preclude long run times and numerical precision [2,5]. Chan and Alfriend posit that in the case of high speed intercepts only the collision angles and the cross-sectional area of the two volumes is of any consequence. Additionally, Alfriend provided the simple mathematics to directly and analytically determine the probability of collision over a range of engagements, varying only the collision angle and cross-sectional area [5]. Using the values from Table 6 and varying the collision angle from 0 to 180, Figure 7 shows the probability of collision as a function of encounter angle. As is clearly depicted there is significant difference between the measurement only solution and the two combined covariance values. This radical variance in probabilites would result in a significant change in planning, scheduling and overall assessment of the RSO's safety of flight as listed in Table 6.

Table 6: 1 Rev Covariance Predictions

1 REV PREDICT	(METERS)		
	RADIAL	IN-TRACK	CROSS-TRACK
MEASUREMENT ONLY	4.65	143.82	5.52
CONSIDER PARAMETERS	38.23	1245.87	49.08
DRAG + CONSIDER	106.67	9758.91	588.83

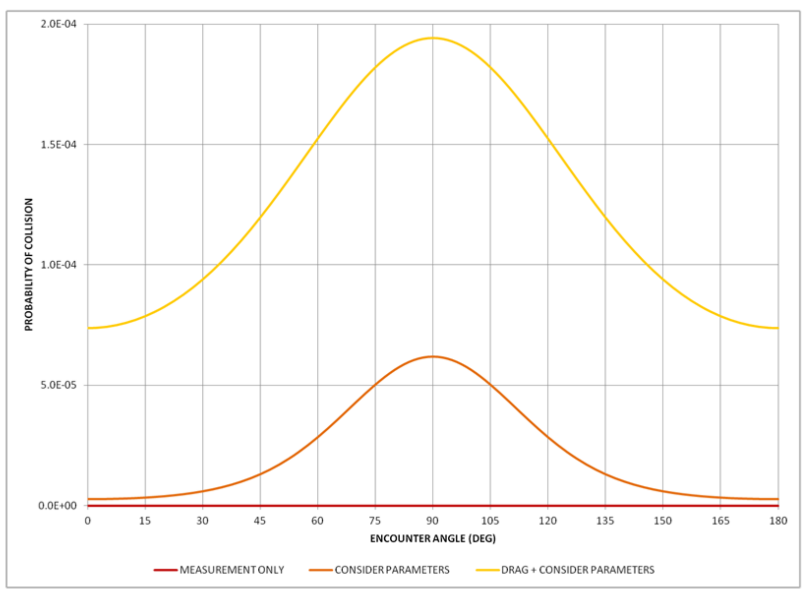

Figure 6: Probabilty of Collision

5.0 SUMMARY

Realistic covariance generation is required to accurately assess the uncertainty in an RSO’s state and their direct impact on probability of collision calculations and safety of flight estimates. The order of magnitude difference between the solved for only and combined covariance increases the risk of collisions and significantly impacts scheduled plans and timelines. To accurately generate the values for the consider matrix, timely and frequent calibration studies must be done to ensure the system’s true performance is reflected in the resulting combined covariance matrix.

References

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