Performance of a Dynamic Algorithm For Processing Uncorrelated Tracks

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Tracks of space objects, which do not correlate, to a known space object are called uncorrelated tracks (UCTs). The association of UCTs to develop an ephemeris, and subsequently a new catalogued object, has typically been a manual process which requires significant time by the analysts. The algorithm used for track association is a static algorithm in that it does not directly take into consideration the uncertainties in the ephemerides determined from the individual tracks. In this paper a previously developed dynamic algorithm based on the track uncertainty (covariance) is evaluated. This dynamic algorithm is based on two new properties: a) the volume of the equiprobability ellipsoid is constant in time, even though the shape changes, and b) the probability of association is maximized by using this ellipsoid for association.

INTRODUCTION

There are currently about 10,000 objects in the Space Catalog whose ephemeris is maintained by USSPACECOM. The sensors of the Space Surveillance System are tasked daily to track these objects. (The NAVSPASUR fence does detect objects but obtains observations, not a track.) When one of the sensors detects an object it checks to determine if the object is in the space object catalog[1]. If it is in the catalog, and if the sensor has been tasked to track it, a set of observations (a track) is obtained. If it is a catalogued object which the sensor has not been tasked to track, then the object is not tracked. If it is not in the catalog, it is tracked. All observations are then passed to the Space Surveillance Center (SSC) denoting whether or not the object is a catalogued object. At the SSC another attempt is made to correlate the track with objects in the space catalog. The association algorithm is a position comparison. The two ephemerides are propagated to the same epoch and a rectangular parallelepiped is constructed about the estimated position in the radial, in-track and out-of-plane directions. If the two objects are in the box then the objects correlate. (There are different degrees of association that will not be discussed here.) If the tracked object does not correlate to any object in the catalog it becomes an uncorrelated track (UCT).

The primary sources of UCTs are breakups, operational satellites that have maneuvered, and small objects that are occasionally tracked. These small objects may be so small that only one radar can detect them, and depending on their orientation on some passes, that radar may not detect them. As a result an ephemeris may not be able to be developed, and the object cannot be entered into the Space Catalog. Consequently, each day numerous UCTs occur. It is then

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necessary to determine which of these tracks associate (correlate). Maintaining the ephemerides of the objects in the Space Catalog is an automated process. However, the processing of the UCTs to determine which ones associate in order to develop an ephemeris has primarily been a manual, time consuming process, which results in a strain on resources. Any improvements in this process would be helpful.

Catalog maintenance could become extremely difficult, if not impossible, if too many UCTs occur. Two situations in which this could occur are:

a) A computer failure, particularly during a time of high solar activity. During this down time the ephemerides of many of the objects in the drag region could degrade to such an extent that tracks of the same object would fail the association test. Each track would then become a UCT. If this occurred with enough objects reconstitution of the catalog might be required.

b) A requirement to track much smaller objects, down close to 1 cm in diameter. This would significantly increase, possibly by an order of magnitude, the number of objects, which had to be tracked on a regular basis, and each track of each new object would become a UCT.

The purpose of this paper is to evaluate a method[2] that will improve the current method of correlating UCTs, and could be used in processing the UCTs in these situations.

In the current UCT association process four assumptions, made about the data,[1] are:

a) Each track of data resulted from observing "something" and is an accurate position for that "something" at the time it was observed. A track consists of both the observations and the element set that best represents the observations.

b) Each element set computed from only one track of data may not be accurate enough to predict a satellite's position within system standards, but the elements themselves are very near the true elements.

c) To consider the orbit determined a minimum of three tracks of data are required on any object, if not solving for drag.

d) Four tracks are required if solving for drag.

The second assumption means that given a number of tracks on the same object there will be a small standard deviation in each individual element about the true value for that object. The standard deviation is not so small as to identify a particular object by merely picking one track elements. However, identifying objects by putting together tracks that have elements that "look" the same is a reasonable concept, and is the first step in the algorithm.

The process is that any track is selected as a reference track and all other tracks are compared to it. In contrast to RETAG[1], which is a position comparison, the UCT association is an orbital element comparison. Let $\mathbf{e} = (e_1, e_2, \ldots, e_6)^T$ be the set of orbital elements. The two UCTs associate if

$$|e_i - e_{0i}| \le \varepsilon_i, i = 1,2,\ldots,6 \tag{1}$$

This is an ordered process of comparison to minimize computation, and the problems of increased uncertainty in some of the orbital elements when the inclination is small. The candidate tracks do not associate if they fail any of the following:

$$\begin{aligned} &|i - i_0| < \varepsilon_1, \varepsilon_1 \text{ nominally 0.3 deg} \\ &\left|\frac{n - n_0}{n_0}\right| < \varepsilon_2, \varepsilon_2 \text{ nominally 0.0075} \\ &|e - e_0| < \varepsilon_3, \varepsilon_3 \text{ nominally 0.008} \end{aligned} \tag{2}$$

If these tests are passed then the angle between the two orbital planes is checked, then the argument of perigee and finally, in some cases, the position in orbit.

The point to be made here is that this is a static association algorithm, it does not take into account the uncertainty or covariance of the track, nor how the element uncertainties change with time. In addition, the in-track error grows with time so the probability of association, Eq. (1), being satisfied for the in-track component, decreases with the time between the tracks. The proposed approach will consider these factors.

DYNAMIC ALGORITHM

Theory

This new dynamic association algorithm takes into account the changing uncertainty of the state variables. Before presenting the algorithm some terms will be defined.

Equiprobability Ellipsoid[3]

Given a system with state $\mathbf{x}$(t), let $\mathbf{C}(t)$ be the covariance matrix, i.e.,

$$\mathbf{C}(t) = E\left(\mathbf{x}\mathbf{x}^T\right),\ E(\mathbf{x}) = 0 \tag{3}$$

where E denotes the expectation operator. Now consider the surface defined by the covariance $\mathbf{C}(t)$, that is the surface defined by

$$S(t) = \mathbf{x}^T\mathbf{C}^{-1}\mathbf{x} = k^2 \tag{4}$$

$S(t)$ is an n dimensional ellipsoid whose semi-axes have length k times the standard deviation of the variable for that axis. This volume is called the equiprobability ellipsoid[3] since the probability density is the same at all points on the surface.

Association Volume

The volume used to associate tracks is called the association volume. For example, the volume described by Eq. (1) is an association volume. The association volume used by USSPACECOM for associating tracks of known objects is a three dimensional box in the in-track, radial and out-of-plane position space centered on the predicted object position. The nominal lengths of the half sides of this box are

$$\begin{aligned} &\text{in track } = \Delta t = 3 \text{ seconds} \\ &\text{radial } = \Delta h = 5 \text{ km} \\ &\text{out of plane } = \Delta\beta = 0.05\,\text{deg} \end{aligned} \tag{5}$$

Some multiple of these values, typically three or four, is used for track association.

To present the new approach it is first necessary to present the two theorems proved in Ref. 2. The following assumptions are made:

- The problem under consideration can be represented by a conservative system, i.e., the forces are derivable from a potential. Except for the atmospheric drag and solar radiation, the external forces on a satellite are derivable from a potential. For track separations of less than a day, which is the problem under consideration, the effects of these two non-conservative forces are usually very small compared to the size of the association volume, and can be ignored except for objects well into the drag region. Such objects will probably decay rapidly.

- The equations of motion can be linearized about the estimated trajectory. Of concern is the deviation of the motion from the reference trajectory defined by the initial state determined from the observations. These deviations, except possibly for the in-track motion, are not large and can be represented by a linear system if the proper coordinate system is used. Ref. 2 shows that the linearized equations represent the motion even when large deviations occur in the in-track direction if the proper variables are chosen.

Theorem 1

For Hamiltonian systems the volume of the equiprobability ellipsoid is constant in time.

Theorem 2

Given an association volume, $S^*(t)$, and an equiprobability ellipsoid, $S(t)$, that have equal volumes, the probability of an object being within the association volume is maximized if the association volume is the equiprobability ellipsoid, that is $S^*(t) = S(t)$.

The first theorem shows that the volume of the equiprobability ellipsoid[3] defined by the covariance matrix is constant in time if all the forces are derivable from a potential. The second theorem proves that for a volume of a given size the probability of being in an association volume is maximized if the association volume is some multiple of the equiprobability ellipsoid. These two theorems establish that the probability of being inside the association volume does not degrade with time as it does with a static position, or orbital element, association volume used for track association. A static association method is currently used for UCT association.

Distribution of *k*

Assuming the probability density function of the state **x**(t) is Gaussian, that is,

$$f_{\mathbf{x}}(\mathbf{x}) = \frac{1}{(2\pi)^{N/2}(\det C)^{1/2}} \exp\left(-\frac{\mathbf{x}^T C^{-1}\mathbf{x}}{2}\right) \tag{6}$$

where C is the covariance then the probability density function and distribution function of k are

$$\begin{aligned} f(k) &= \frac{1}{2^{(N/2-1)}\Gamma(N/2)} z^{N-1} \exp\left(-k^2/2\right) \\ F(k) &= \frac{1}{2^{(N/2-1)}\Gamma(N/2)} \int_0^k r^{N-1} \exp\left(-r^2/2\right) dr \end{aligned} \tag{7}$$

where $\Gamma(x)$ is the Gamma function defined by

$$\Gamma(x) = \int_0^\infty u^{x-1} \exp(-u) du \qquad (8).$$

For $N = 6$

$$f(k) = \frac{1}{8} k^5 \exp\left(-k^2 / 2\right)$$
$$F(k) = 1 - \frac{1}{8}\left(k^4 + 4k^2 + 8\right)\exp\left(-k^2 / 2\right) \quad (9)$$

Figure 1 shows the theoretical probability density function *f(k)* and the density function from numerical experiments. To perform the numerical experiment six sets of 1500 random

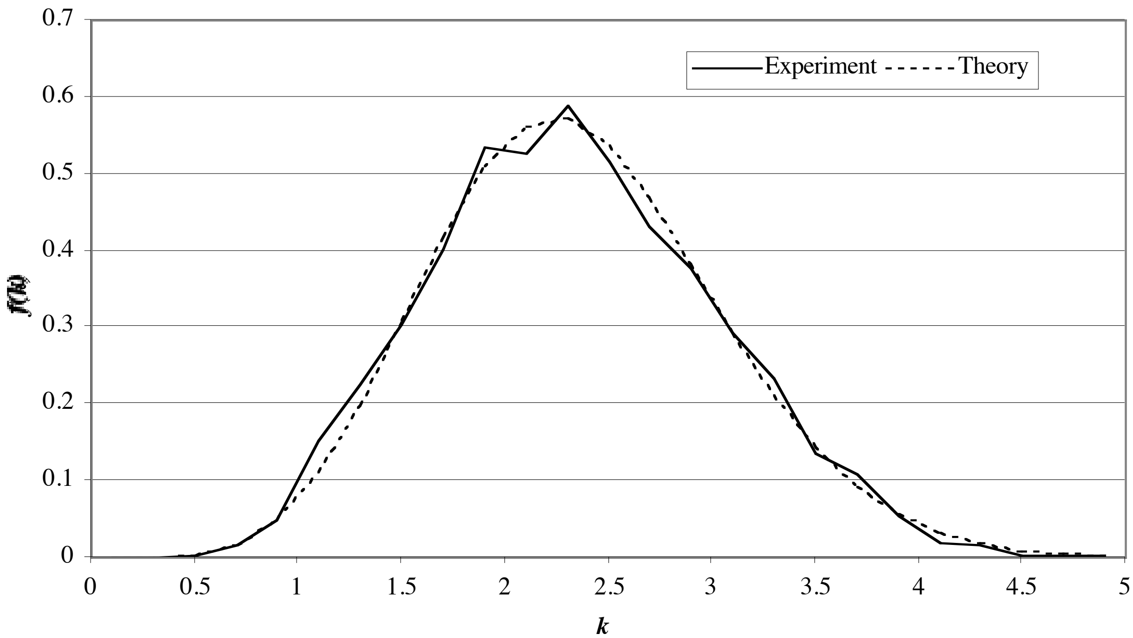

Figure 1 Theoretical Probability Density Function *f(k)* for $N = 6$

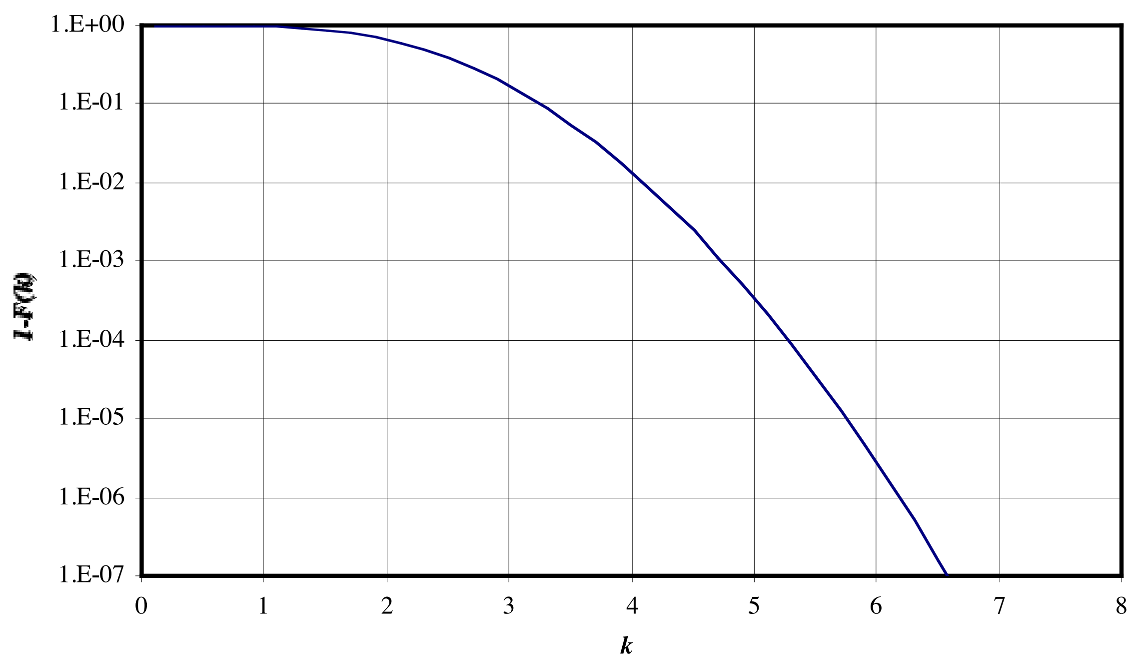

Figure 2 Complement of the Distribution Function *F(k)*

numbers with Gaussian distributions with different variances were generated and the functions $f(k)$ was computed. Figure 2 shows the complement of the distribution function $(1-F(k))$. From the distribution function one can select a value of *k* to use to ensure from a theoretical basis a high probability of associating the tracks. Of course, as *k* increases so does the probability of false associations. These two concepts have to be balanced against each other. The size of the covariance is also a factor in the process.

NUMERICAL EXPERIMENTS

First the concept was validated with simulations. A satellite was placed in a near circular orbit with a=6800 km. and i=51.6 deg. A sensor was located at Eglin and true observations at intervals of five seconds for two minutes were obtained for each pass through the radar. The observations were than corrupted with Gaussian white noise with standard deviations equal to the published figures for the FPS-85 at Eglin. The force model was J_2 with no atmospheric drag. The orbit was obtained for each track using the standard least squares differential correction approach. The state and covariance were then propagated to the epoch of the next track. Denoting these two tracks by subscripts j=1,2 and the state by $\mathbf{x}_j$, compute *k* for each pair of tracks via

$$\begin{aligned} \mathbf{z} &= \mathbf{x}_2(t_2) - \mathbf{x}_1(t_2) \\ C(t_2) &= C_1(t_2) + C_2(t_2) \\ k^2 &= \mathbf{z}^T C^{-1} \mathbf{z} \end{aligned} \tag{10}$$

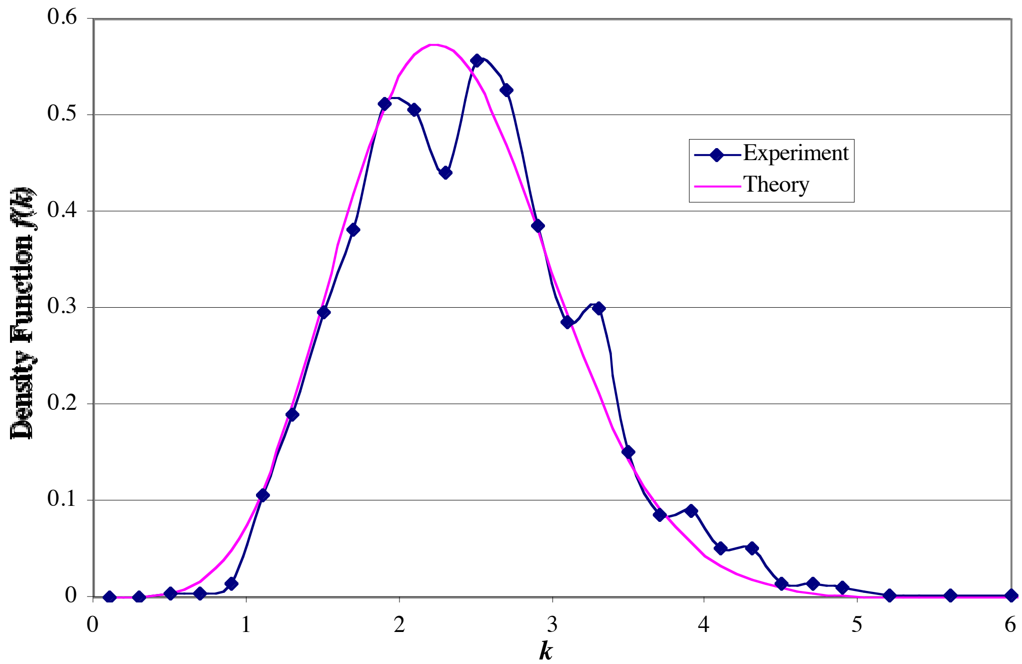

Figure 3 Probability Density Function Comparison – Numerical Experiment

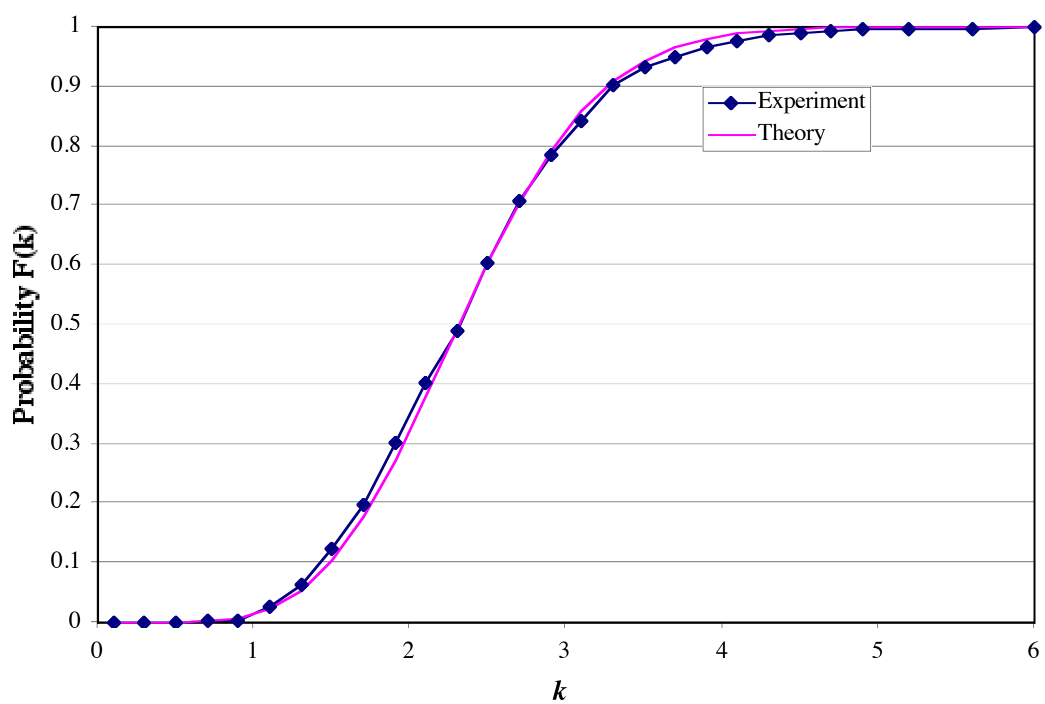

Figure 4 Probability Distribution Function Comparison – Numerical Experiment

Figures 3 and 4 compare the theoretical and numerical density and distribution functions from 500 trials. It is evident that the agreement is excellent and the numerical experiment validates the theory.

Next, to determine the performance of the algorithm, two sets of numerical experiments with on-orbit data were performed. The sets of non-operational objects in low Earth orbit (LEO) for these experiments are given in Table 1.

Table 1
Satellites Used in Numerical Study

	Object No.	Name	Perigee (km)	Apogee (km)	Inclination(deg)
Set 1	5398	Gridsphere	756	860	101
	10702	Landsat 3	894	917	98.8
	13259	Cosmos 1375	988	1001	65.8
	13736	OPS 9845	797	809	98.6
	13777	IRAS	885	903	99
Set 2	6155	OAO-3	660	725	35
	20496	Lace	452	466	43
	22076	Topex	1331	1342	66

For each object each track is considered to be a UCT. The ephemeris and covariance for each track are obtained from the LSDC process with epoch as the time of the first observation. The ephemeris and covariance are then propagated to the epoch of the next track that is within 24 hours. With the error being the difference in the two states and the covariance the sum of the covariances of the two tracks the value of k is calculated using Eq. (4). Treating these values of k as a random variable the distribution is obtained and compared with the theoretical distribution given by Eq. (9). In addition, for each pair of tracks Eq. (2) is used to determine if the two tracks correlate using the current static algorithm. For the satellites in Set 1 almost all tracks had only three observations. This is not indicative of a UCT. Bad data and sensor biases will have much more effect on state errors for these short tracks. The results from Set 1 are presented in Figures 5 and 6. Although the peak of the distribution is coincident with the theoretical peak there are many large values of k. About 20% of the cases had values of $k > 10$. Table 2 shows the number of track comparisons with $k > 8$, $k > 12$ and $k > 16$. Also shown are the number that would have been rejected using the portion of the static algorithm given in Eq. (2).

Table 2

Sat #	# of tracks	# k >8	# k >12	# k >16	Static alg.
5398	61	7	4	2	3
10702	100	47	36	31	44
13259	95	15	12	10	20
13736	95	18	14	12	27
13777	41	22	17	13	22
Total	392	87	66	55	116

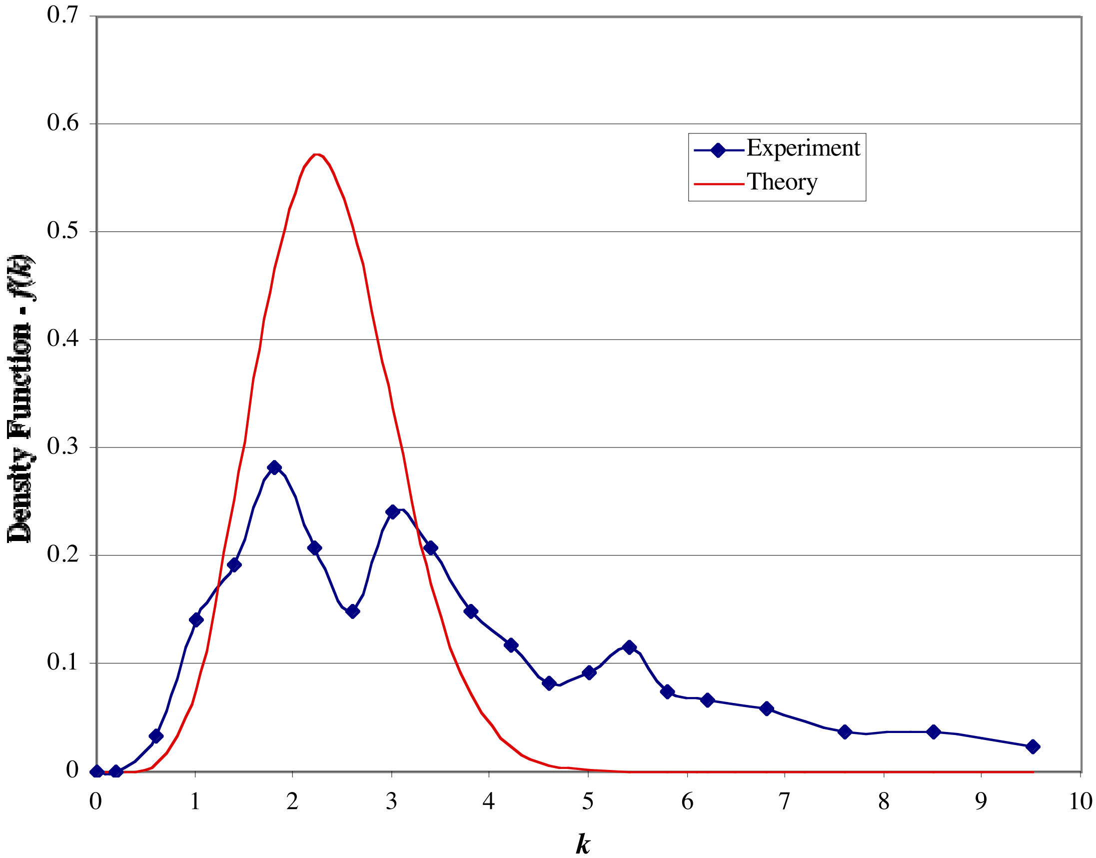

Figure 5 Probability Density Function for Set 1

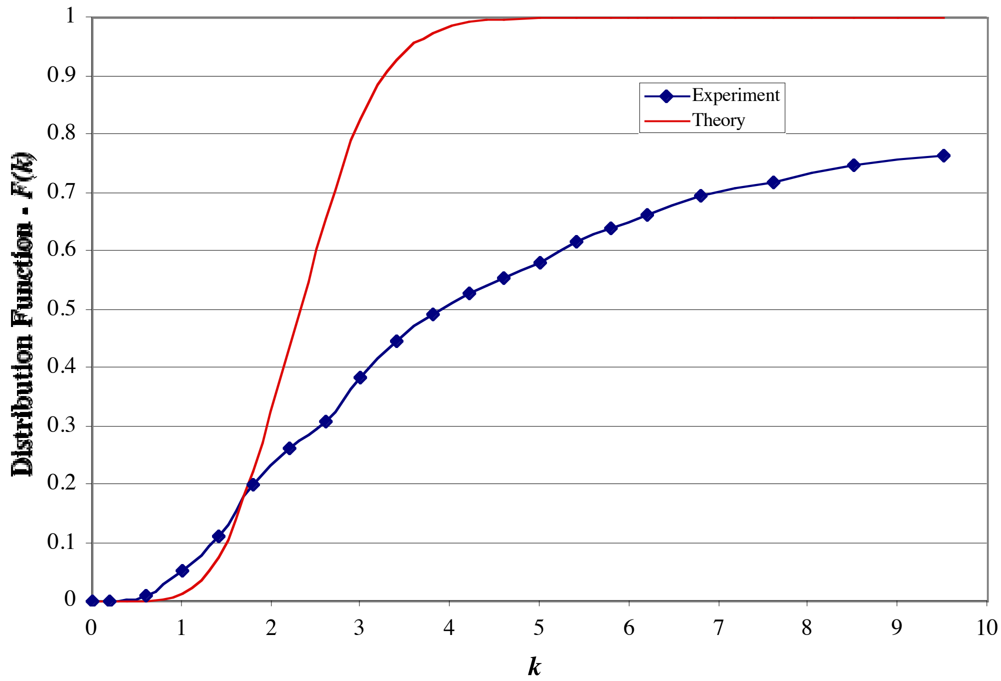

Figure 6 Distribution Function for Set 1

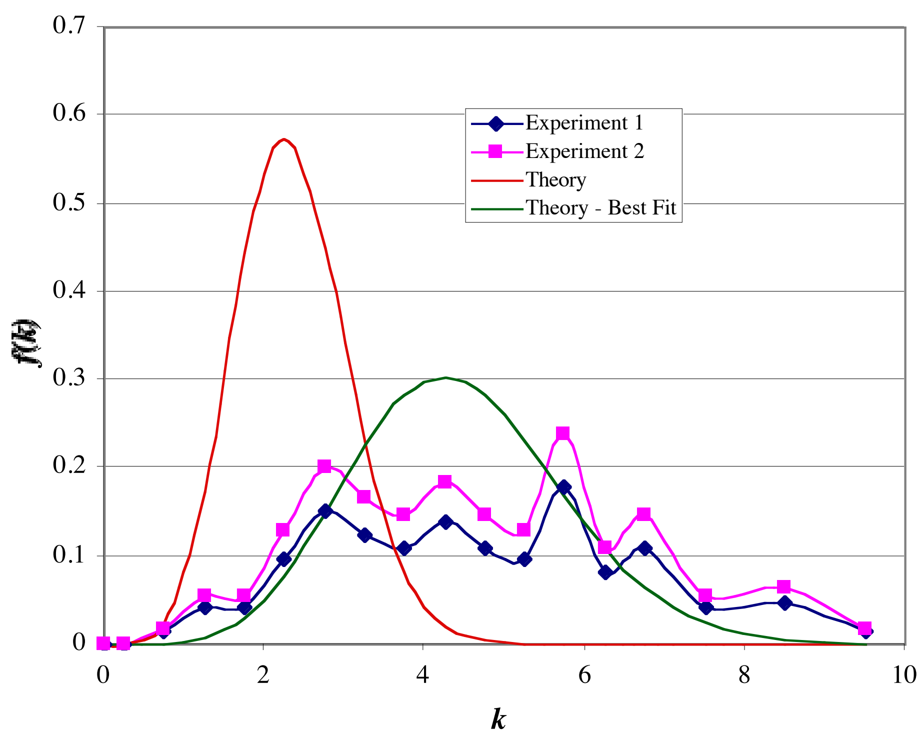

Figure 7 Probability Density Function for Set 2

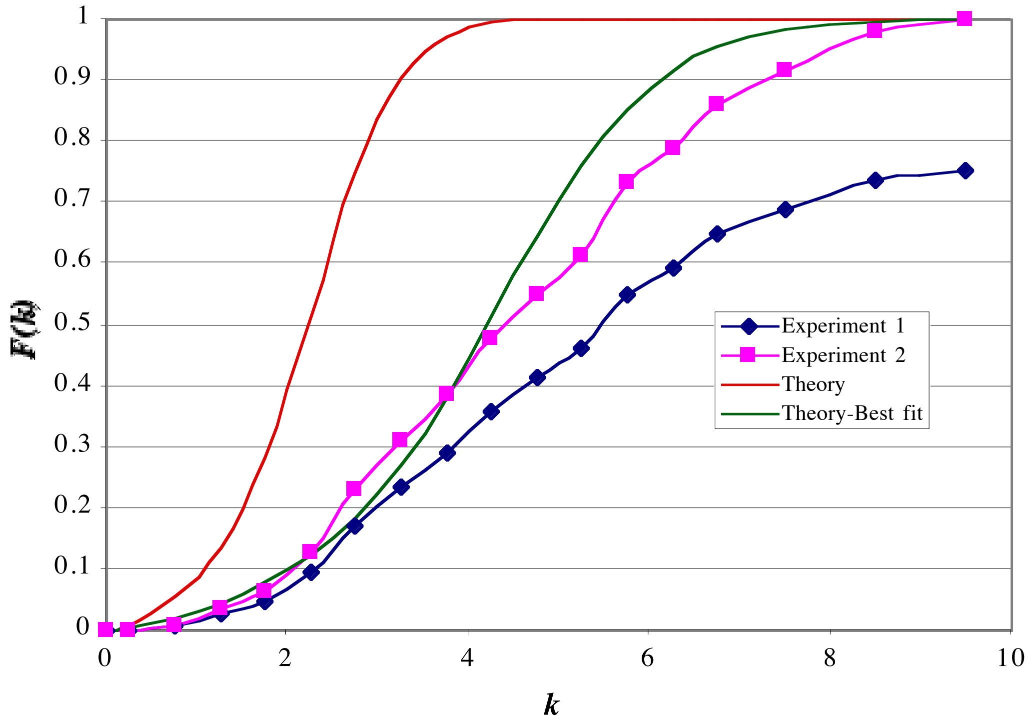

Figure 8 Distribution Function – Set 2

Figures 7 and 8 show the probability density function and distribution function for Set 2. The data for the satellites in Set 2 are the data used in Ref. 4 by Vallado in his study of short arc dense tracking. Observations were separated by one second and the average number of observations per track was approximately 100. This is much more indicative of the type of data for UCTs than the data in Set 1. Shown are two curves for the orbit data and two for the theoretical. The curves shown are for all values of k and all values of $k < 10$. The best-fit theoretical curve is obtained by assuming all sensor noise values are in error by the same factor and then finding by least squares the error factor. The value obtained was 1.89. Both sets of satellites had approximately the same percentage of values of $k > 10$. Since there were many more observations in each track this is indicative of incorrect modeling of the sensor error characteristics. These data were obtained just several weeks before the workshop and more extensive investigations are planned.

CONCLUSIONS

A new dynamic algorithm for UCT processing has been proposed, validated theoretically by simulation and applied to on-orbit data. Satellite Set 1 consisted almost entirely of tracks of only three observations whereas Satellite Set 2 had tracks of approximately two minutes and 100 observations. Performance against both sets was similar. In both sets approximately 22% of the values of k were $k > 10$. This many large values of k is indicative of incorrectly modeled sensor errors, noise sigmas and/or biases. Force model errors would not cause these large values of *k* for single tracks and propagation times of a few orbits. The data of Set 2 were obtained just shortly before the conference so a more extensive investigation is planned.

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