Comparative Analysis of Different Methods for Revealing Latent Periodicities in Processes

S. Veniaminov, V. Dicky, A. Shevyrev
SRC "Kosmos", Moscow, Russia

Some cases of incorrectness and delusion still frequently being met in the practice of revealing latent periodicities in signals and different processes are discussed and analyzed methodically on some special illustrative examples and real physical processes. Some dangerous consequences of such wrongness in the real practice of processing radar and optical signatures are considered and a correct approach to the problem is suggested.

When researching the multiple periodical structure of different physical, sociological, business, and other processes, there exists the faulty practice of using the Fourier transformation for revealing latent periodicities in the processes concerned. However, transforming the process in such a way, the investigator forgets that Fourier analysis is intended as a matter of fact for a very different task and that its essence is alien to the problem of revealing latent periodicities in signals and processes.

At the same time, in some cases Fourier analysis can be successfully put into practice for revealing latent periodicities in some classes of processes, though it should be used very cautiously and guardedly. One should be sure that the process concerned meets some hard requirements and restrictions.

So, for a scientifically grounded start to the investigation of the process with the help of Fourier analysis, the investigator should have certain rather specific information on the process. In particular, one should be sure that the measurement interval is many times as long as the longest period of the periodic component sought and that all the components are pure harmonic ones.

The fact is that the principal parameter of the Fourier transformation (that is the measurement interval length) has nothing to do with the periods of periodical components sought . Nevertheless, it is used for forming the functional basis for approximation of the process concerned.

So, the problem being solved by Fourier analysis in essence differs from our problem – that is the problem of revealing latent periodicities. As the sequence, for example, the harmonics with the periods more than one half the measurement interval length would not even be "felt" by Fourier analysis. That means that the application of the harmonic analysis to our problem is counter-indicative for some ratios of the harmonic period and the measurement interval length.

Besides, the harmonic analysis by its essence comes to the approximation of any process in the most inappropriate and unnatural to our problem functional basis – the set of series of namely harmonics having their periods as whole divisors of the measurement interval length. That means that the harmonic analysis is absolutely inapplicable in the case of revealing nonharmonic periodicities. For example, instead of one pure rectangular pulse component (which can be, say, the optical signature of a mirror surface of some satellites) Fourier analysis gives several false harmonics with doubtful unauthentic periods.

Finding. The problems of the best approximation of the processes and that of revealing latent periodicities have essentially different goals.

As distinct from the harmonic analysis, there exist a large set of methods worked up especially for revealing latent periodicities [1, 2, 5]. As some researches show [2, 3, 4, 5], among them the so called D-method and K-method [2, 5] mostly fit for solving our problem. They stand up against noise and distortion of the periodicities form. The more so, these methods reveal components with very close periods and any correlation of the periods and the measurement interval length. It is preferable only that the last should be at least a little more than the largest period.

And now we come to the short description of the methods.

Generally, D-method [2, 4] comprises a whole family of methods of different orders. It is a transformation of the initial process given by the following expression:

$$D_x(\tau) = \frac{1}{\bar{t} - \tau} \int_0^{\bar{t}-\tau} \left| \zeta^n(t,\tau) \right| dt,$$

where $\zeta^n(t,\tau) = X^n(t+\tau) - X^n(t)$;

$X(t)$ is the process concerned given at the interval $[0;\ \bar{t}]$;

$X^n(t)$ – the n-th derivative of $X(t)$;

τ - an auxiliary variable of time dimension.

K-method uses the generalized correlation transformation of the process concerned:

$$K^n(\tau) = \frac{1}{\bar{t} - \tau} \int_0^{\bar{t}-\tau} X^n(t) X^n(t+\tau) dt.$$

In both methods the order of the derivative *n* determines the order of the method and can essentially affect its resolution possibility. However, one can control the specific properties of each method not only by choosing its order but also by the recurrence of transformations – a number of successive applications of the transformation at first to the initial process, then to its *D*-image or *K*-image, then to the result of the second transformation, and so on.

Namely this system of control levers for the *D*- and *K*-methods gains the powerful tool kit for analysis of the most inconvenient processes.

The low order methods are more resistible to the influence of noise, and reliably reveal the long period components. The increase of the order enhances the sensibility of methods to the high frequency components of the process.

The increase of the recurrence of *D*- and *K*-transformations on the contrary suppresses the high frequency components (including noise) and helps reveal the long period components.

Now address the analytical estimate of the *D*-mapping characteristics in presence of nois in the discrete task set.

Let the errors of the measuring process $X = \{X_k\}$ be Gaussian with the zero expectation an r.m.s. be σ_x. The discrete multiple harmonic process looks like

$$X_k = \sum_{i=1}^{I} A_i \sin\left(\frac{2\pi k}{T_i} + \varphi_i\right) + \xi_k, \qquad k = 0\,..\,N-1, \tag{1}$$

where N – number of measurements,
A_i – amplitude of *i*-th harmonic,
T_i – period of *i*-th harmonic,
φ_i – phase of *i*-th harmonic,
ξ_k – error of *k*-th measurement,
I – number of harmonics.

At the outset consider the D_0-image of the process. In discrete mode we will have

$$D_0(k) = \frac{\sum_{j=0}^{N-k} \left| X_{j+k} - X_j + \xi_{j+k} - \xi_j \right|}{N-k},$$

or

$$D_0(k) \le \hat{D}_0(k) + \frac{\sum_{j=0}^{N-k} \left|\eta_j\right|}{N-k}, \quad k = 0\,..\,N-1,$$

where $\hat{D}_0(k)$ is *D*-image of the noiseless process,
$\eta_j = \xi_{j+k} - \xi_j$ - Gaussian random variable.

The distribution of / η_j / has the next parameters:

$$m_{|\eta|} = \frac{2\sigma_x}{\sqrt{\pi}}, \qquad \sigma^2_{|\eta|} = \left(2 - \frac{4}{\pi}\right)\sigma_x^2.$$

The error of D_0-image is limited from above by a quantity

$$\Delta D_0(k) = D_0(k) - \hat{D}_0(k) \le \frac{\sum_{j=0}^{N-k} |\eta_j|}{N-k},$$

and its expectation – by $1.1\sigma_x$.

The variance of the error of D_0-image is limited from above by a quantity

$$\sigma^2[\Delta D_0(k)] \le \frac{(2-\frac{4}{\pi})\sigma_x^2}{N-k} \approx \frac{0.7\sigma_x^2}{N-k}.$$

Hence, the presence of noise in measurements leads to the bias of *D*-image. The amount of the bias is proportional to the r.m.s. error of measurements. The variance of the estimate of *D*-image is proportional to the noise variance and inversely proportional to the measurement interval length.

The bias of the estimate of *D*-image does not influence the accuracy of the period determination, because it is not along the abscissa axis, but normal to that, and for this family of methods only the local minima determination accuracy is important, that is their location at the abscissa axis.

The accuracy of the determination of the periods of the harmonics, and moreover, the possibility of such a selection is wholly determined by the variance of the *n*-th order, which can be assessed as

$$\sigma^2[\Delta D_n(k)] \le \frac{2\sigma_x^2(1-\frac{2}{\pi}) \sum_{i=0}^{n} (C_n^i)^2}{N-k-n}, \quad \mathrm{k = 0 .. N - n - 1}.$$

The dependence of the accuracy of period determination on the noise intensity, the *D*-mapping order, the harmonic's amplitude and period can be derived as follows.

For *D*-method of *n*-th order:

$$D_n(k) = 2A\left(\frac{2\pi}{T}\right)^n \left|\sin(\frac{\pi k}{T})\right| * \frac{\sum_{j=0}^{N-n-k} \left|\cos(\frac{2\pi(j+k)}{T}) + \varphi\right|}{N-n-k} \approx \frac{2^{n+2} A\pi^{n-1}}{T^n} * \left|\sin(\frac{\pi k}{T})\right|.$$

At the vicinity of local minima *D*-image has its first derivative by time (by *k* in discrete case), with the accuracy up to the sign, equal to

$$\frac{dD_n(k)}{dk} = \frac{2^{n+2}\pi^n A}{T^{n+1}}.$$

Then, the estimate of the r.m. s. error of the local minimum abscissa value determination is

$$\sigma_n(k) = \sqrt{\frac{(1-\frac{2}{\pi})\sum_{i=0}^{n}(C_n^i)^2}{N-n-k}} * \frac{T^{n+1}}{2^{n+1.5}A\pi^n}.$$

Hence, for specified levels of noise and signal it is possible to select the harmonics only given the definite length of the measured time interval. If its span is less than some “critical” value, then the family of “pure” *D*-methods does not fit, to say nothing of all traditional methods. At the same time, a radar or optical signature usually contains an intensive noise component.

To overcome the shortcomings of the known methods, on the basis of selective *D*-methods and autocorrelation functions, the new family of methods for revealing latent periodicities were developed for different levels of noise.

Further, from the examples of special artificial and real processes, one can see the comparative effectiveness of *D*- and *K*-methods and Fourier analysis. The comparison of the *D*-method with some other methods one can found in [2, 3].

At Fig.1 a sample of the simplest ideal process is given. It is less than one and a half period of a pure sinusoid with period T = 300c and amplitude A = 1. As can be seen from Fig.2, Fourier analysis cannot solve the problem of revealing the single harmonic. Meanwhile, *D*-method (Fig. 3) determines its period with absolute accuracy.

The next example (Fig. 4) – the same sinusoid with additive noise (r.m.s. of noise amplitude equals a half signal amplitude). The presence of noise somewhat complicates the task. Fourier analysis does not work as before (Fig. 5). *D*-method solves the task with no difficulty (Fig. 6).

Fig. 7 shows the same intelligence signal (T = 300 s, A = 1) and the noise is one and a half times more. Fourier analysis fails as was expected. Single *D*-transformation and *K*-transformation fail as well (Fig. 8). But reiterated *D*-transformation and *K*-transformation (recurrence equals 2) determine the harmonic period exactly enough (Figs 9 and 10). A special procedure was elaborated for refining the period of harmonic as well as its amplitude and r.m.s. of the noise (Fig. 11).

The next example (Fig. 12) – a pure harmonic (T = 300 s, A = 1) given at the interval of 20 times its period length. In this case Fourier analysis works pretty well (Fig. 13) as well as *D*- and *K*-methods.

At Fig. 14 there is the same intelligence signal, but with intensive noise (r.m.s. = 1). Fourier transformation reveals the intelligence signal (Fig. 15) but does not answer whether there are other periodicities or not. At the same time, *D*-method (Fig. 16) solves the problem unambiguously and correctly.

Fig. 17 gives more complex example – 3 harmonics ($A_1 = 2$, $A_2 = 1$, $A_3 = 4$, $T_1 = 47$, $T_2 = 62$, $T_3 = 54$). Fourier transformation reveals only 2 of them and with great errors (Fig. 18). *D*- and *K*-methods reveal all 3 harmonics exactly enough (Figs 19 – 22).

Fig. 23 – two very close (by their frequencies) harmonics ($T_1 = 40$ s, $T_2 = 41$ s). Here Fourier analysis is absolutely insolvent and unsound (Fig. 24). However *D*- and *K*-methods cope with this task well (Figs 25 – 27).

Fig. 28. The intelligence signal consisting of 3 close harmonics nearly lost in the noise, which is comparable by its amplitude with the highest harmonic and excels the next two (A_1=1, $A_2 = 2$, $A_3 = 3$, $T_1 = 10$ s, $T_2 = 12$ s, $T_3 = 15$ s, noise r.m.s. = 3). This case is very hard for analysis but a very real one. Fourier transformation can cope with this task (Fig. 29) only at the very long measurement interval including 700 – 1000 oscillations of the highest harmonic (that is 7000 s – 10000 s). As can be seen (Figs 30 – 32) *D*-method has no problem. To obtain this luxurious registration in practice is usually difficult of access and rather expensive. If we have only the 100 s measurement interval (100 times less), then Fourier analysis absolutely fails (Fig. 33) while *D*-method works rather well (Figs 34 – 36).

One more very important and didactic example (Figs 38 – 41). The process is the sum of two, not harmonic but rectangular, pulse periodic components ($T_1 = 0.8$ s, $T_2 = 2.4$ s). *D*-method reveals them with no problem. However, the result of Fourier analysis should be considered very attentively. In this case Fourier transformation gives 6 false components instead of 2 real ones and so brings the investigator into delusion.

And now the last (but not less didactic) example of the process containing, besides two periodical components (T_1=10s, T_2 = 17s), a non-periodical one – a square parabola (Fig. 42). Fig. 43 shows that this non-periodical component diminishes the sensitivity of Fourier-method. So it reveals no periodical components, while D-method solves the problem perfectly (Figs 44 and 45).

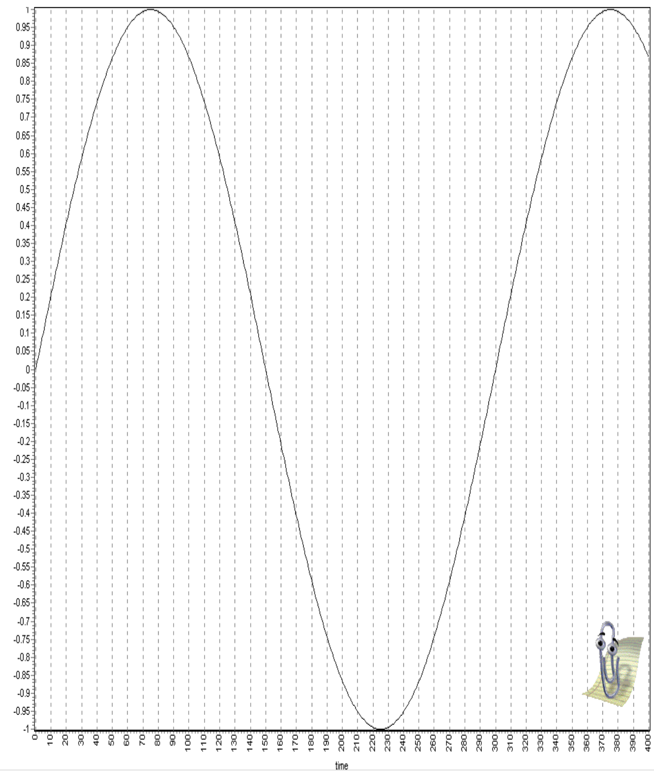


Fig. 1. Canonical example

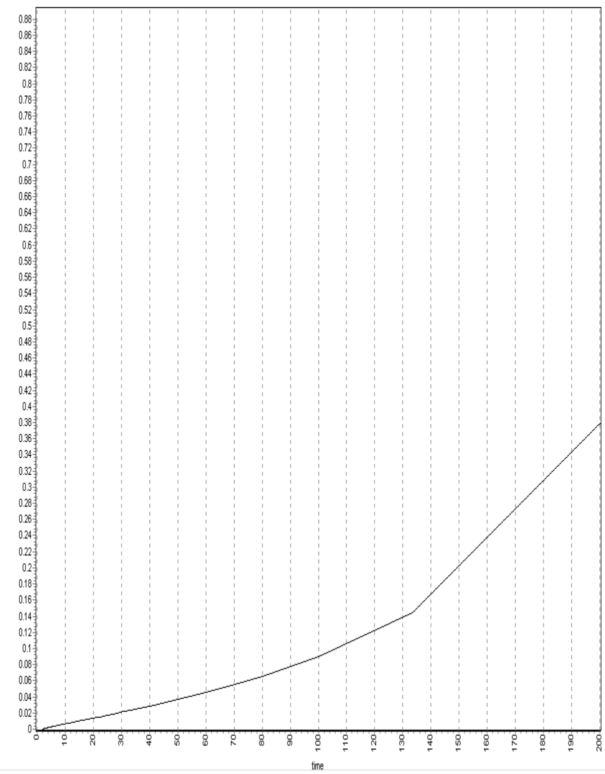


Fig.2. Fourier transformation failure

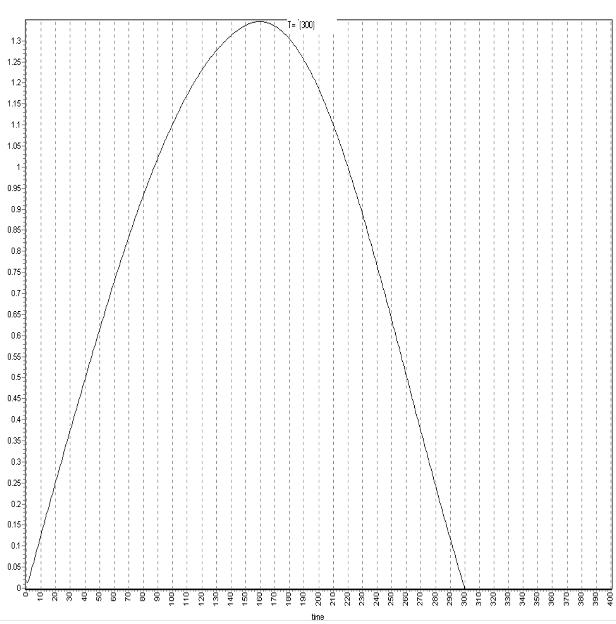


Fig.3. **D-*transformation***

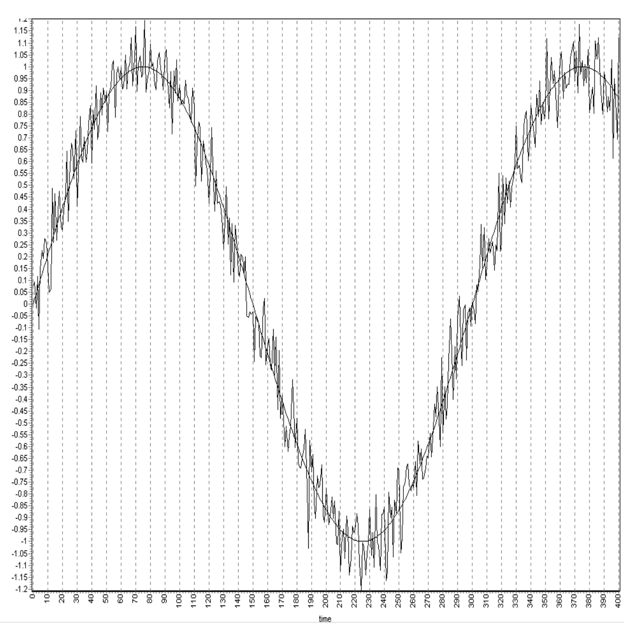


Fig. 4. Noisy sinusoid

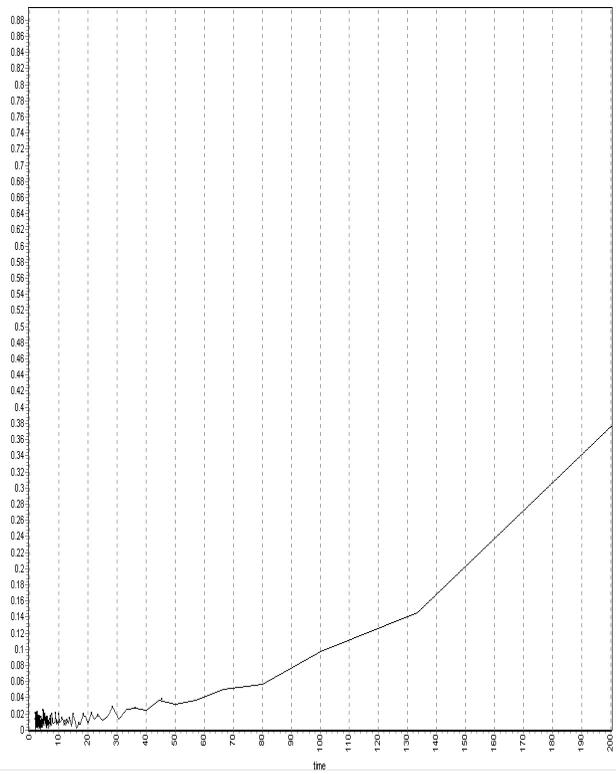


Fig.5. Fourier transformation failure

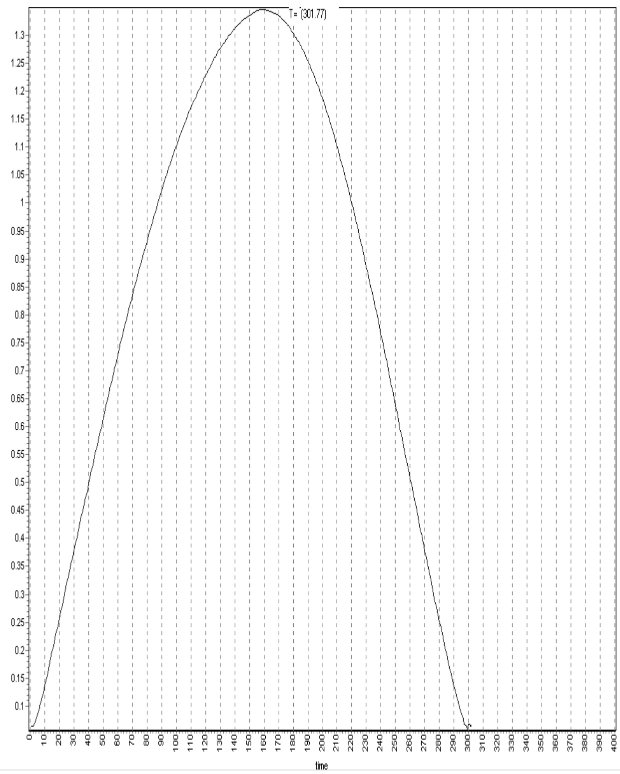


Fig. 6. *D*-transformation

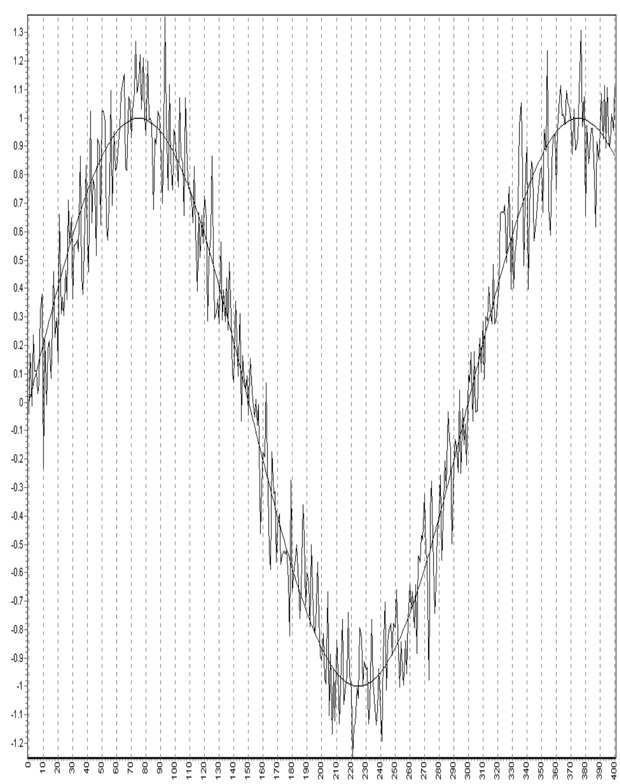


Fig. 7. More intensive noise

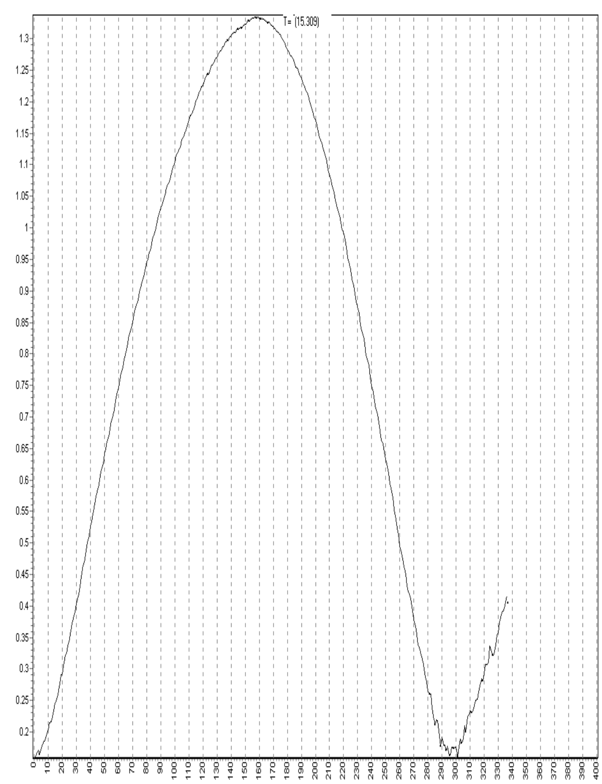


Fig.8. *D*-transformation of 1st order

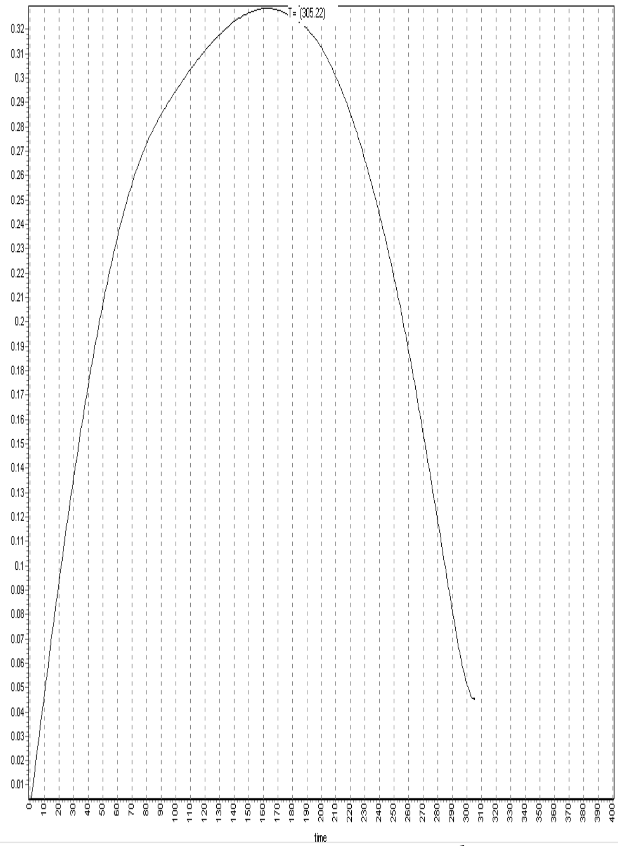


Fig. 9. **D**-*transformation of* 2^{nd} *order*

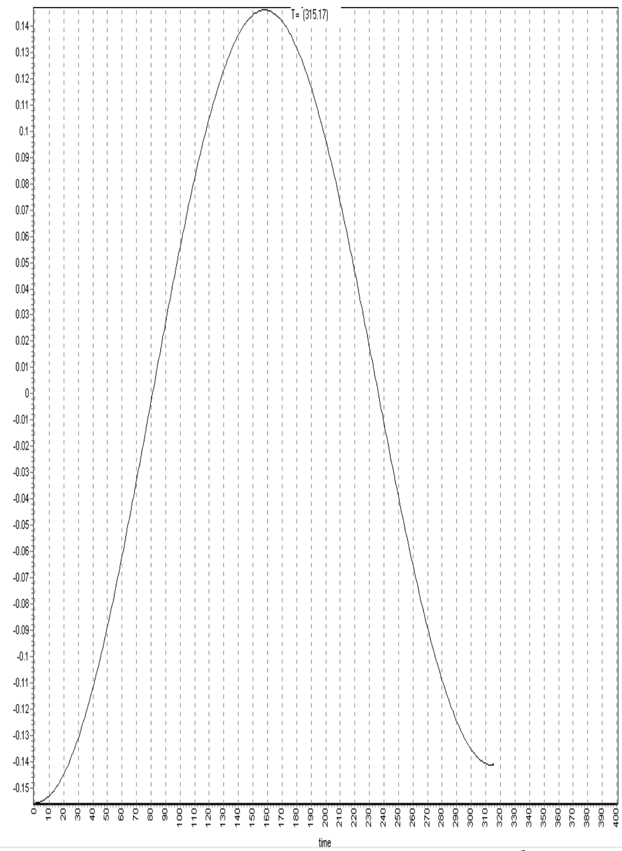


Fig. 10. **K**-*transformation of* 2^{nd} *order*

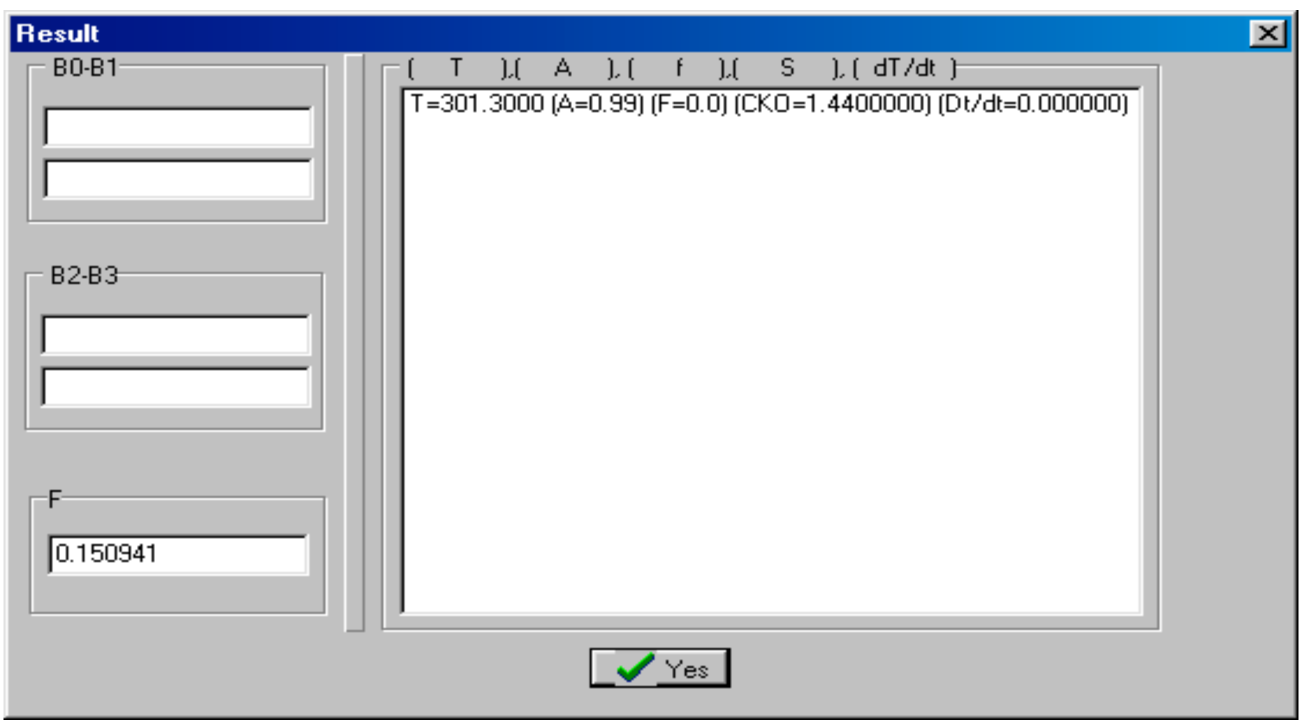
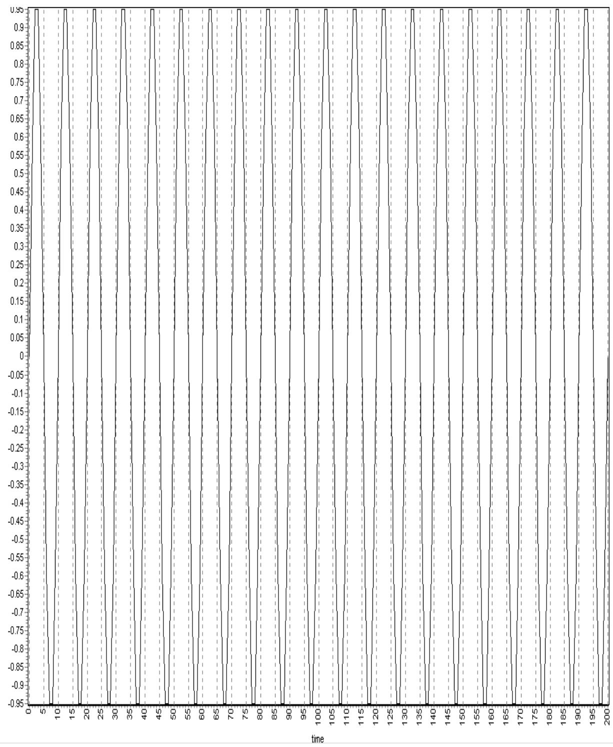


Fig. 11. Special refining procedure summary result



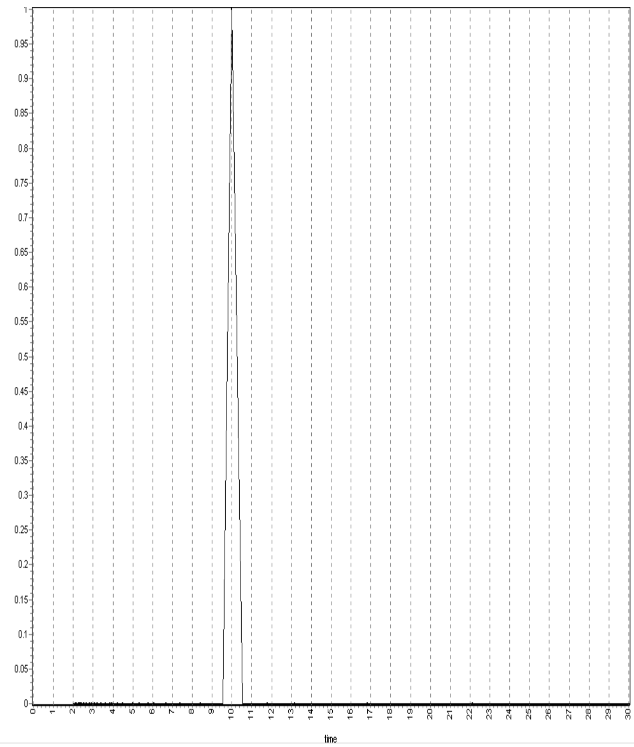
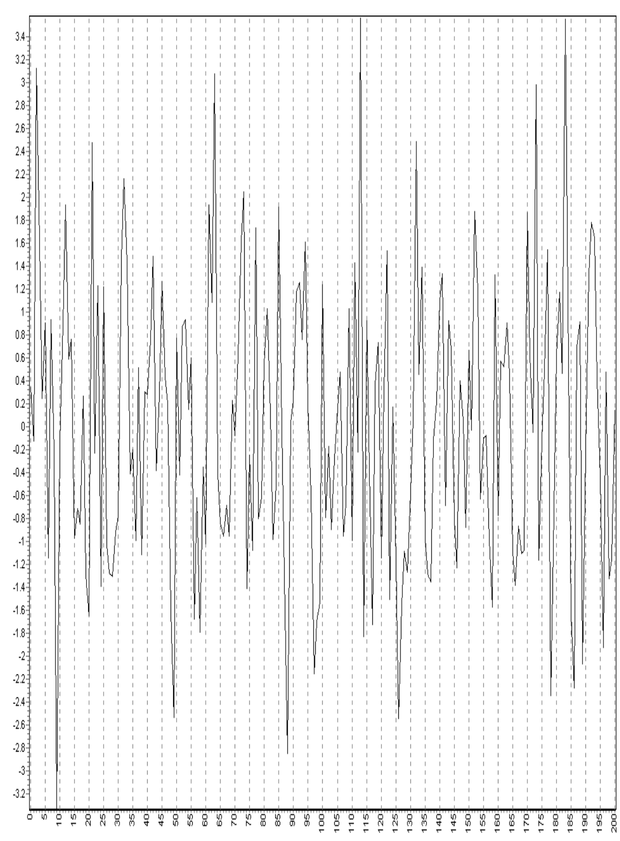


Fig. 12. Another canonical example Fig.13. Correct Fourier result



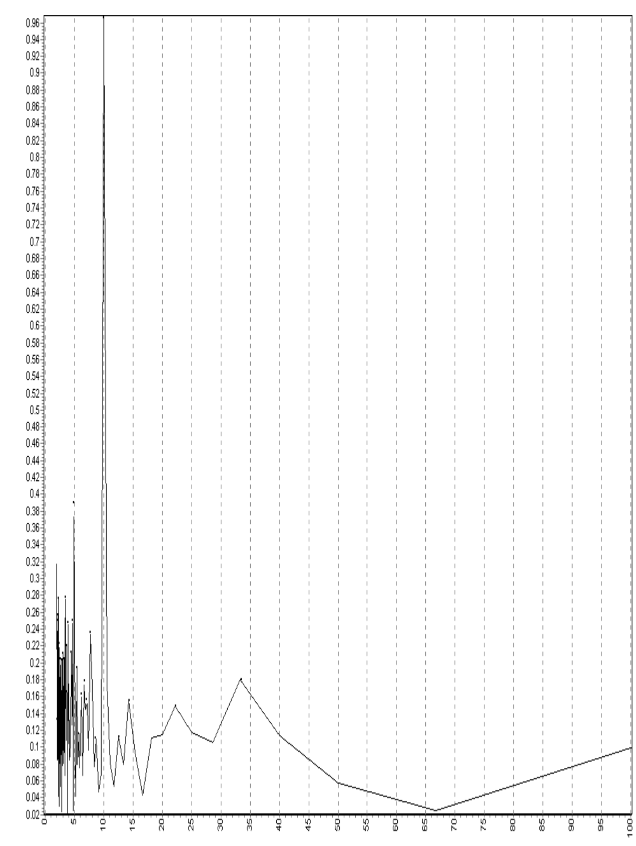


Fig. 14 Very noisy signal Fig. 15. Ambiguity of Fourier analysis

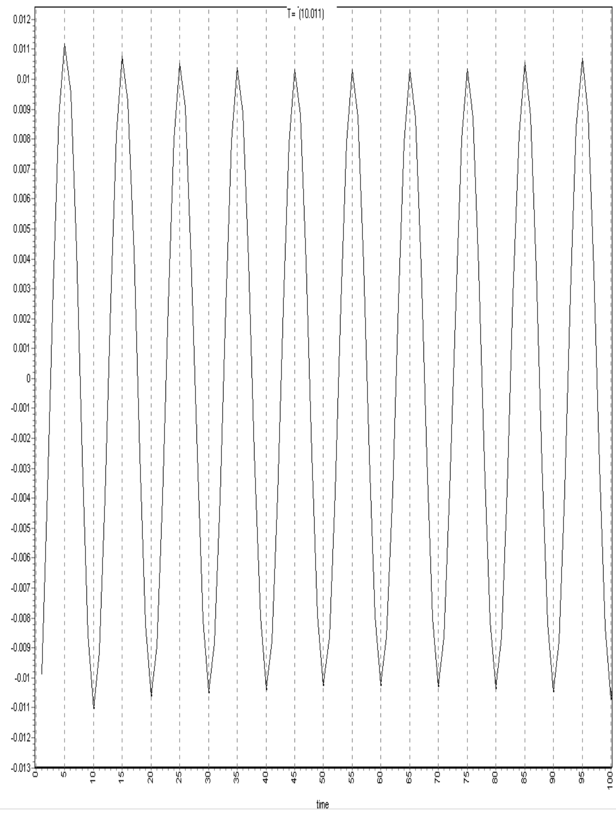


Fig.16.*D*-transf. giving unambiguous solution

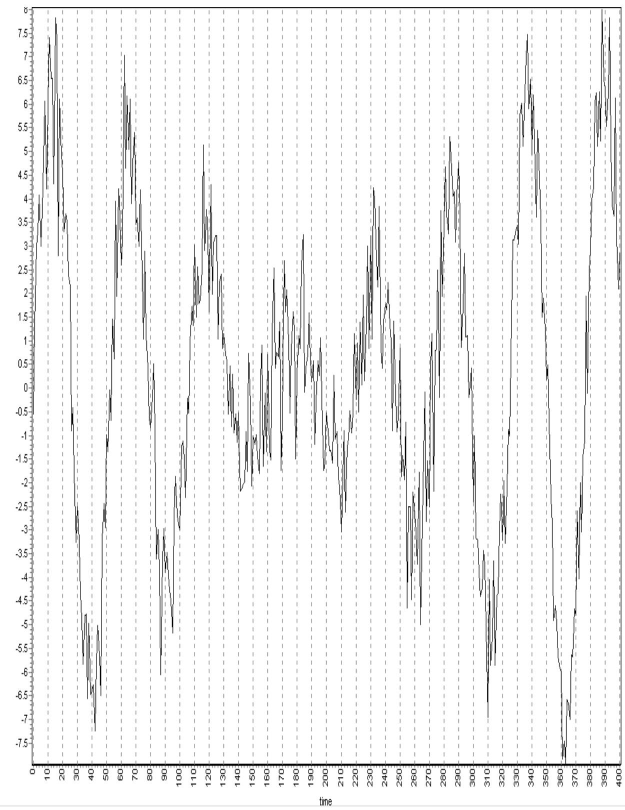


Fig.17. Three noisy harmonics

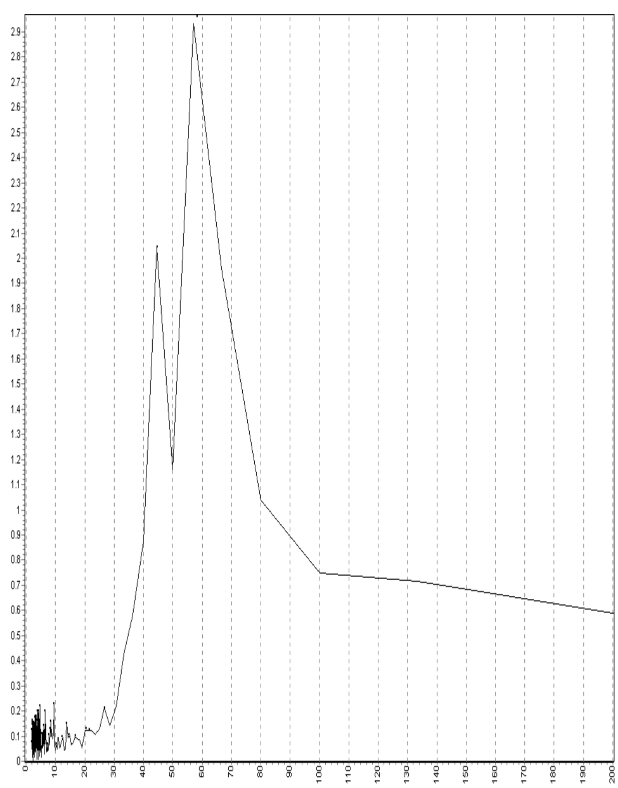


Fig. 18. Fourier transformation failure

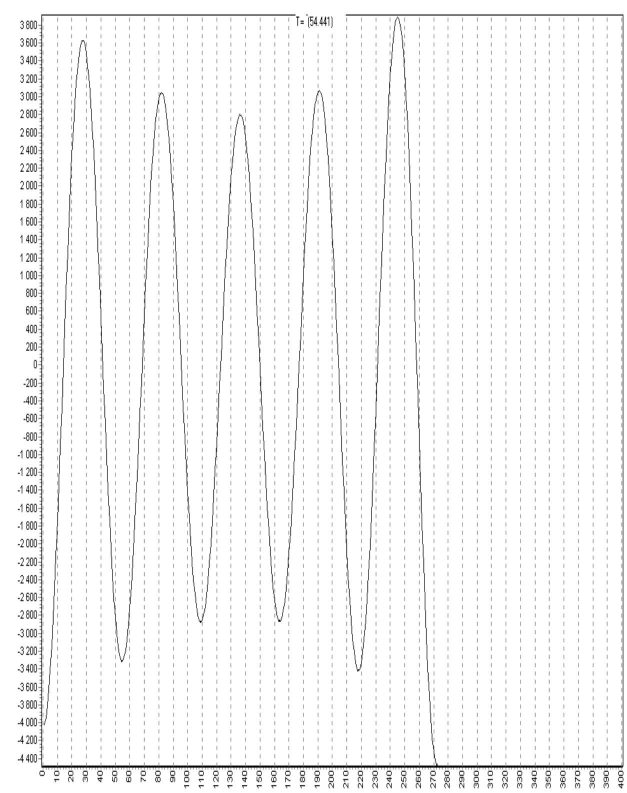


Fig. 19. 2^{nd} harmonic (*D*-transform.)

Fig. 20. 1st harmonic (D-transform.)

Fig. 21. 3d harmonic (D-transf.)

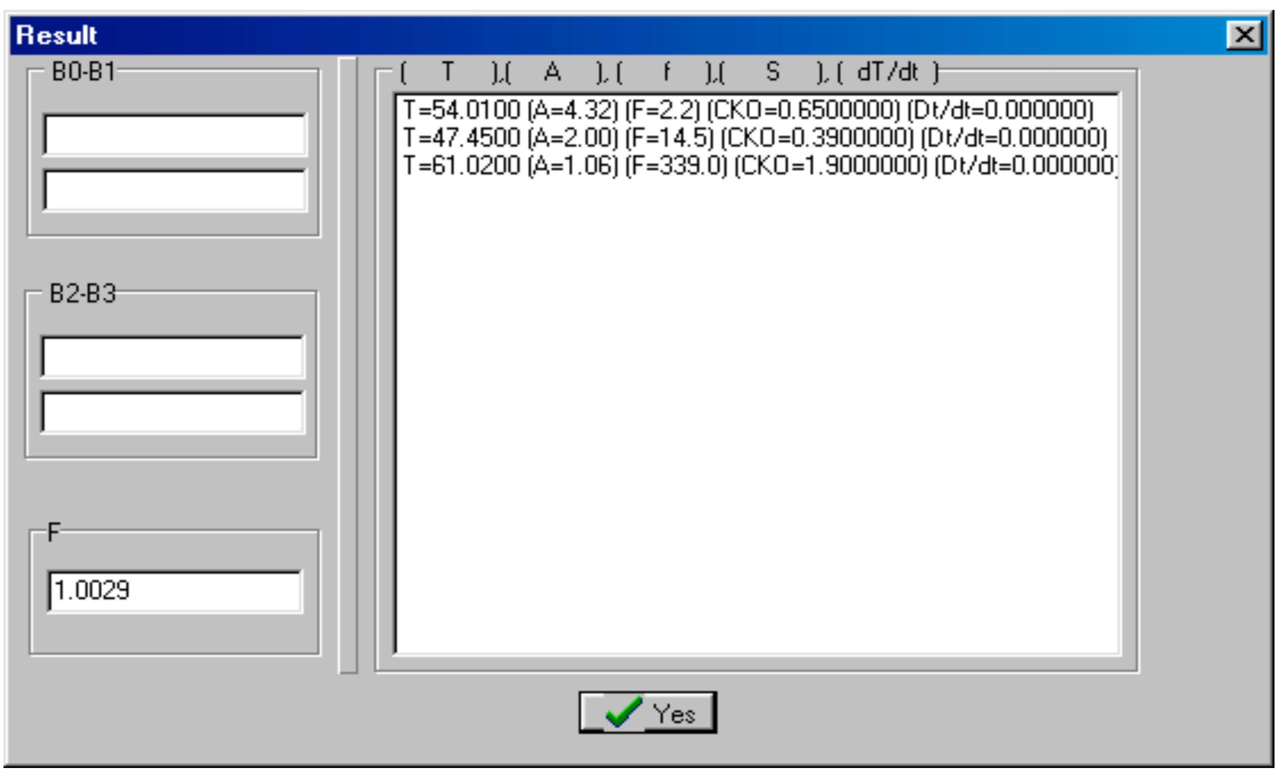


Fig. 22. Special refining procedure summary result

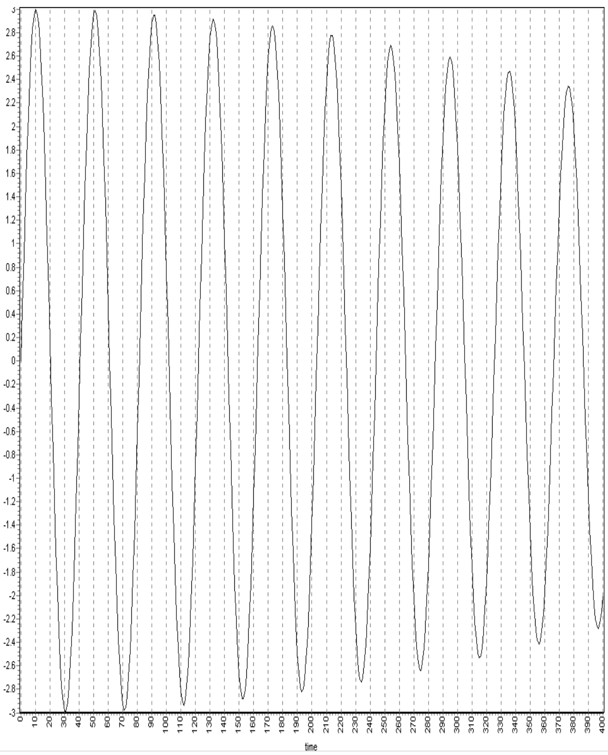


Fig. 23.Sum of 2 very close harmonics

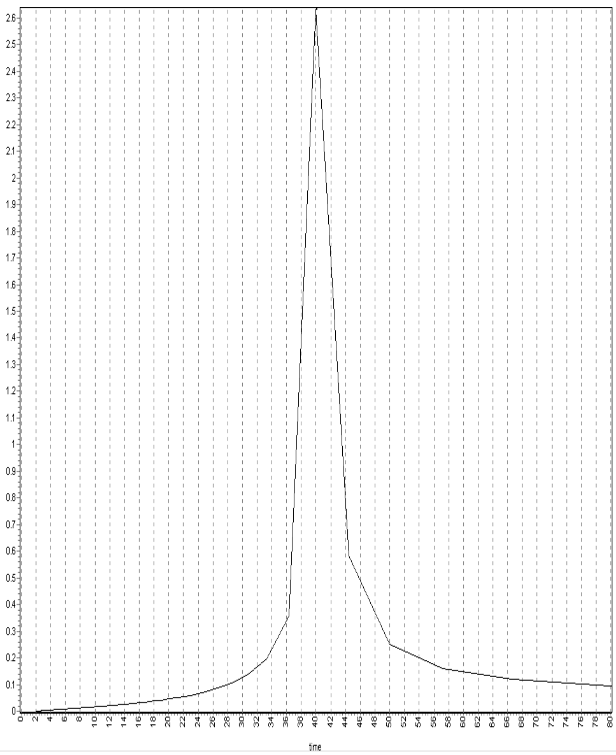


Fig.24. False Fourier result

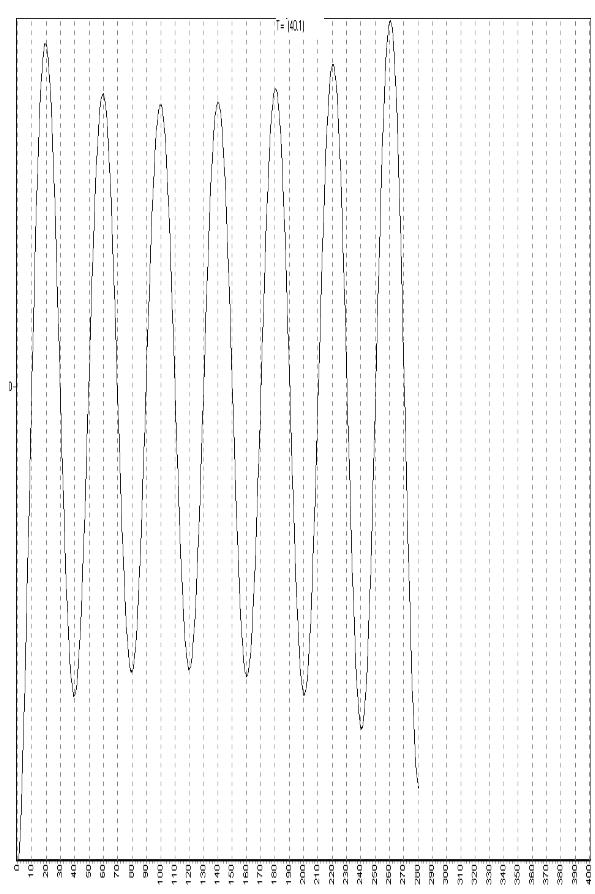


Fig.25.* D-*transf. revealing 1st harmonic

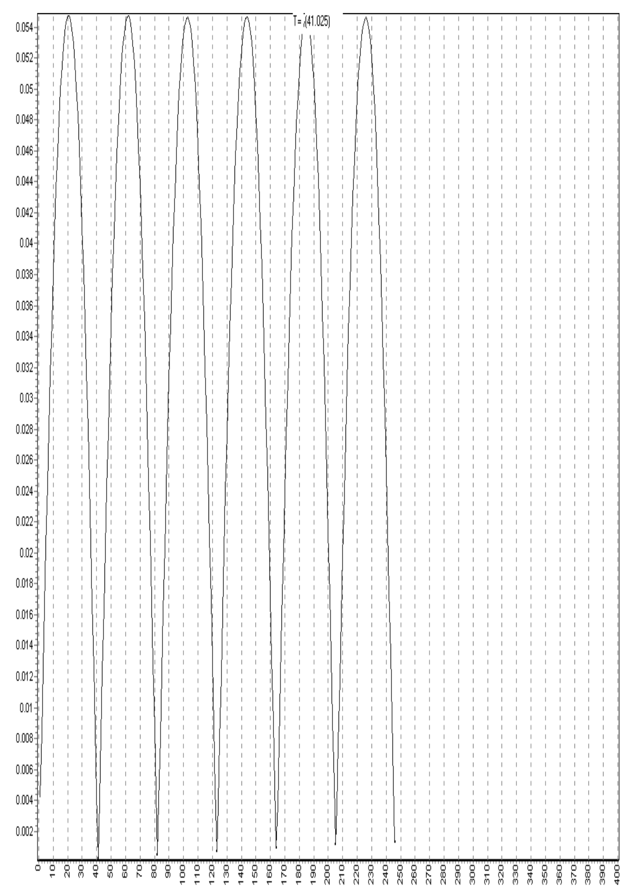


Fig.26.* D-*transf. revealing 2nd harmonic

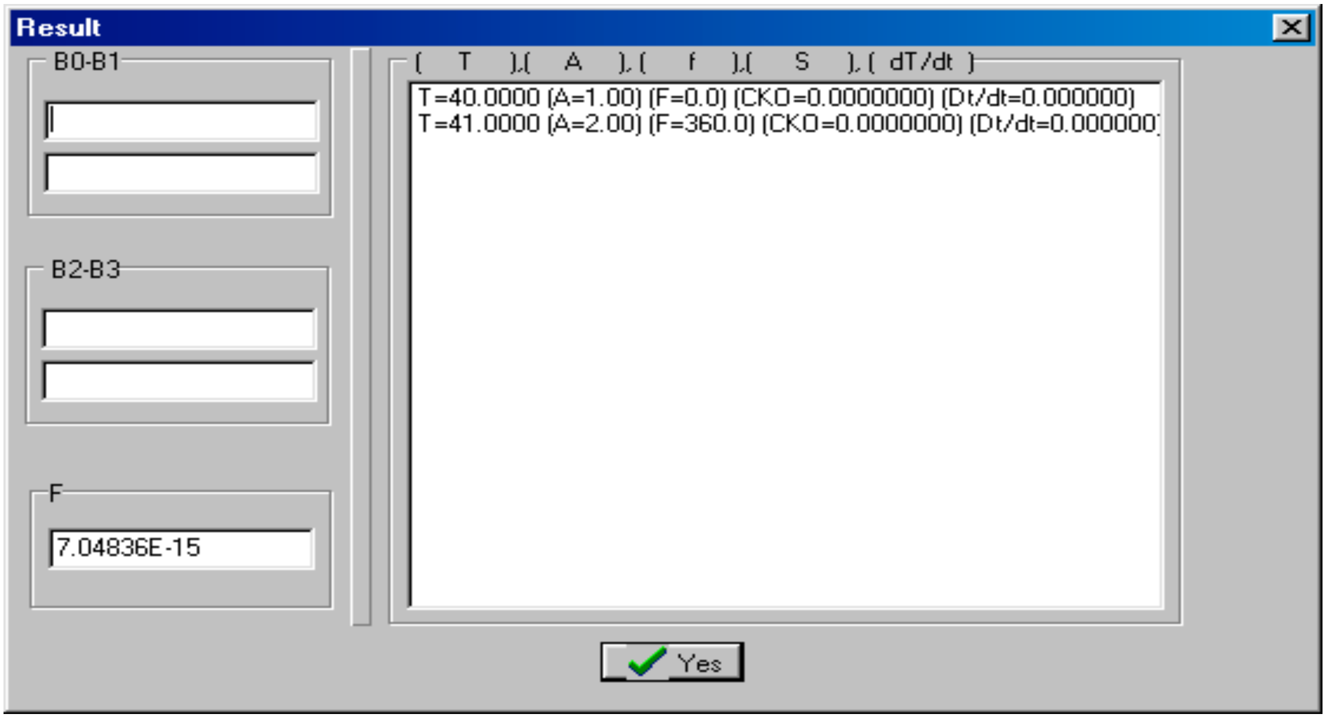


Fig. 27. Summary result

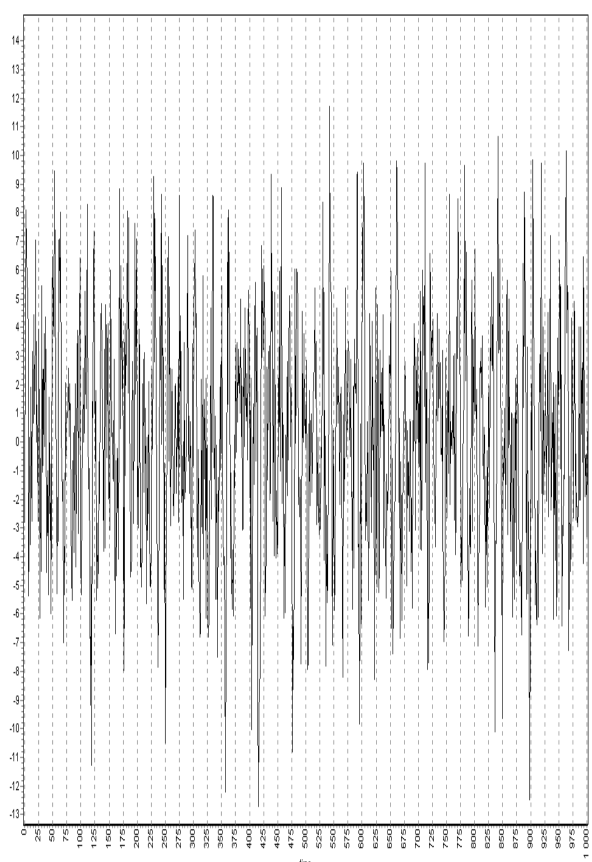


Fig.28. 3 very close harmonics and noise

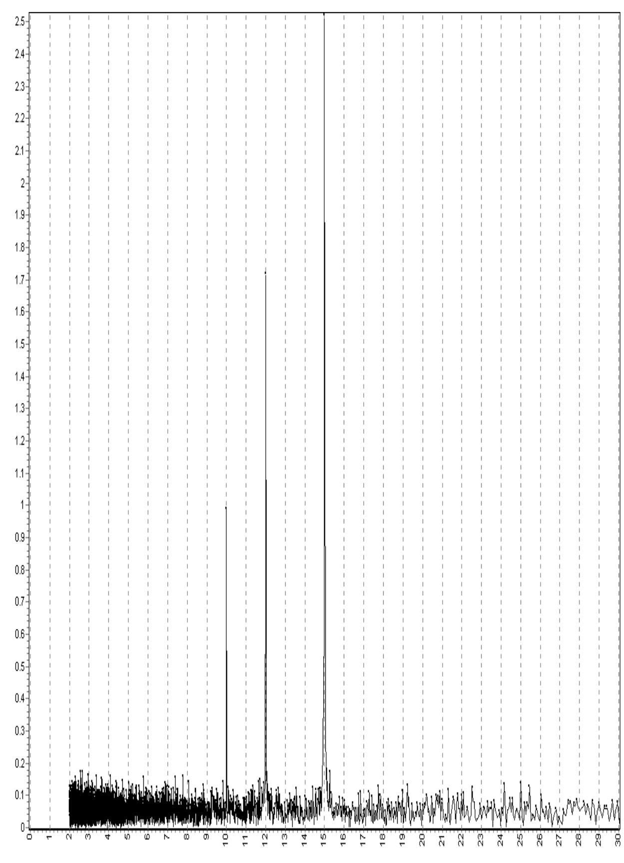


Fig.29. Fourier transf. (meas.int.10 000s)

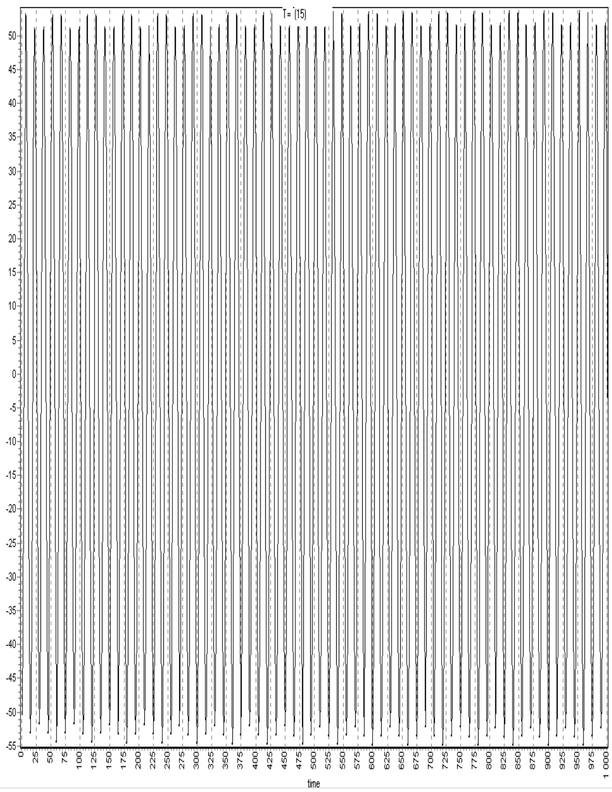


Fig.30.*D*-transformation revealing T=15s

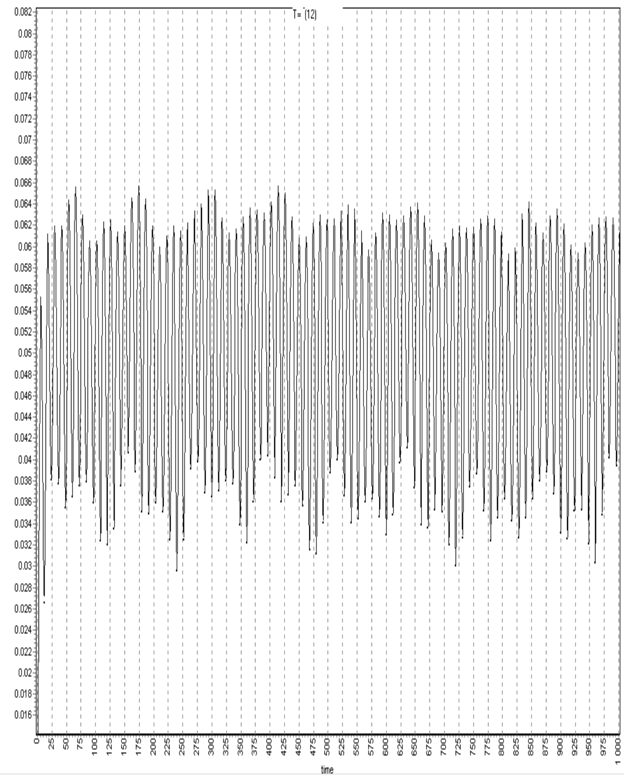


Fig.31. *D*-transformation revealing T=12s

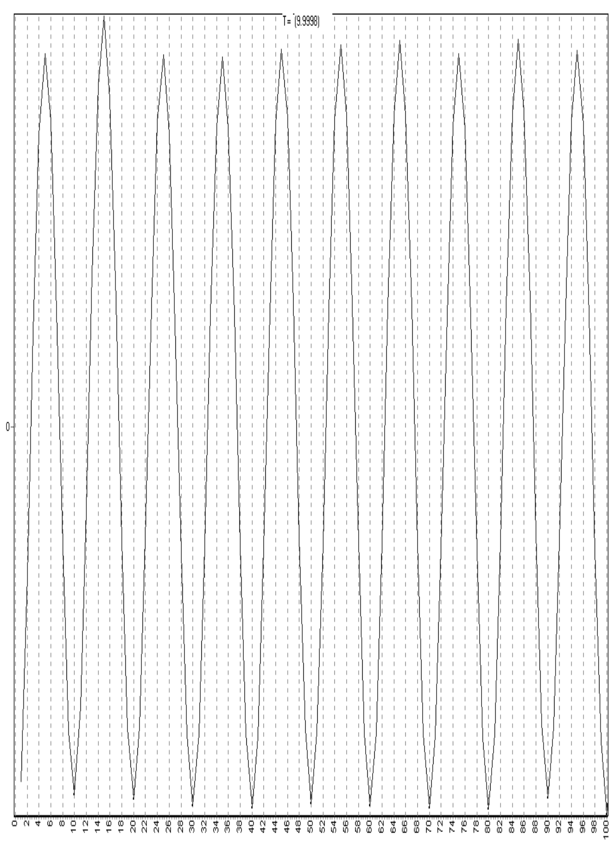


Fig.32.*D*-transformation revealing T=10s

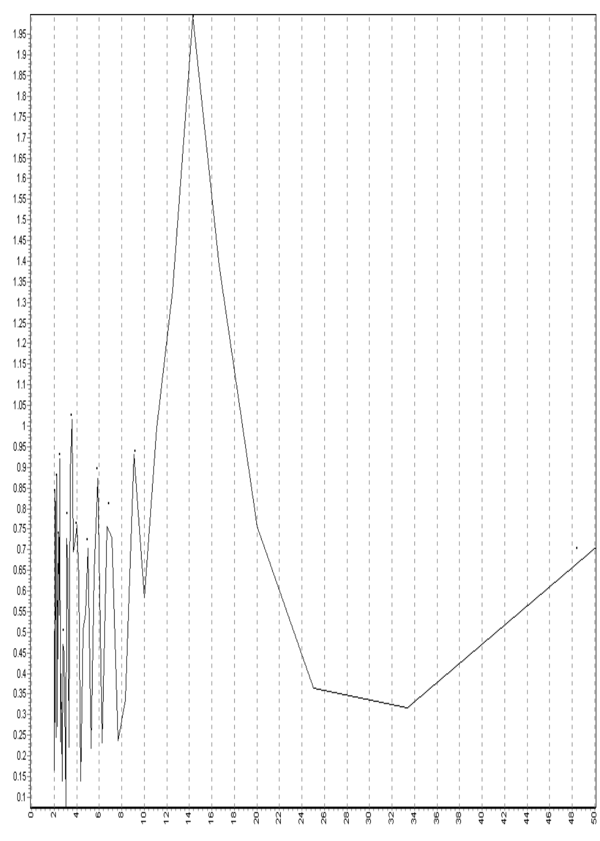


Fig.33.No Fourier solution (mes.int.100s)

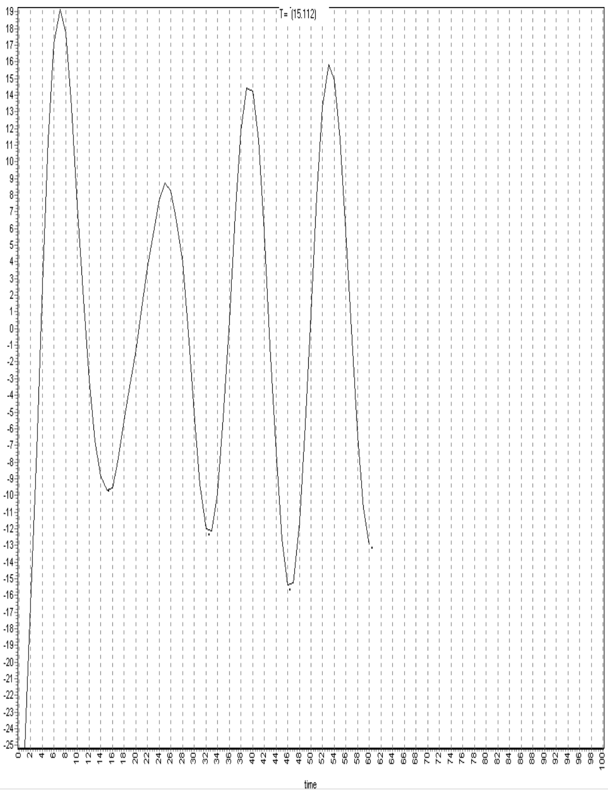


Fig.34.*D*-method gives T=15s (m.int.100s)

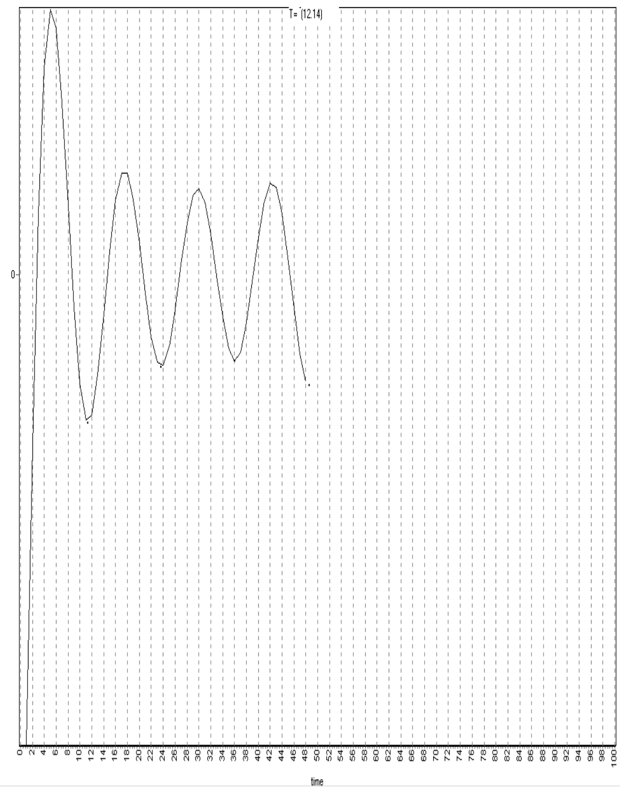


Fig.35.*D*-method gives T=12s (mes.int.100s)

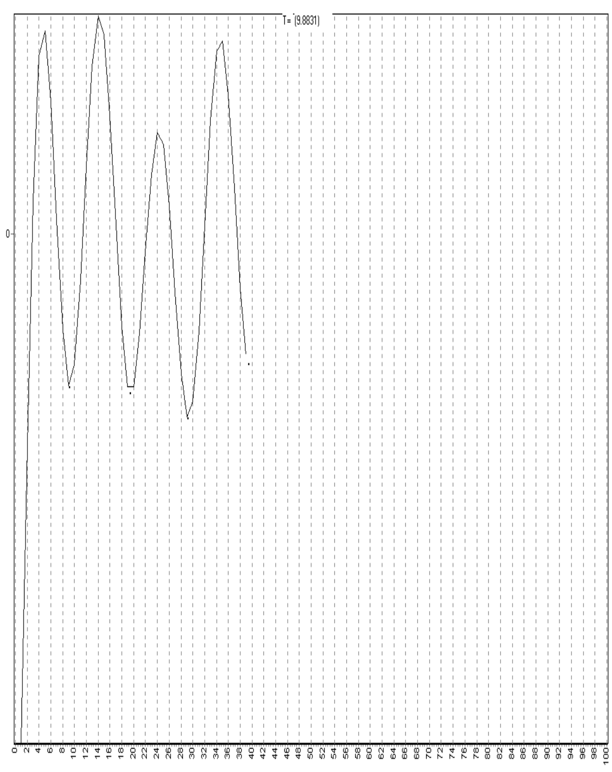


Fig.36.*D*-method gives T=10s (m.int.100s)

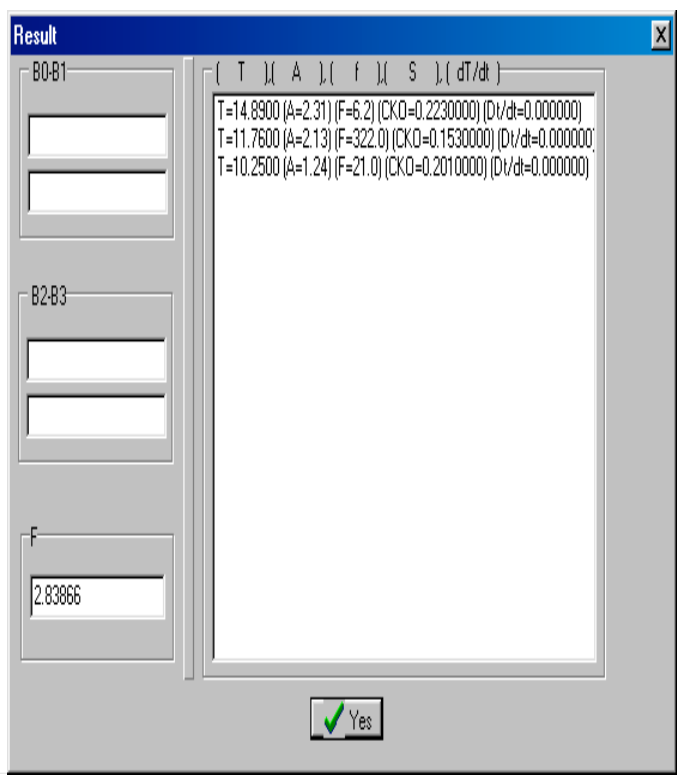


Fig. 37. Summary result

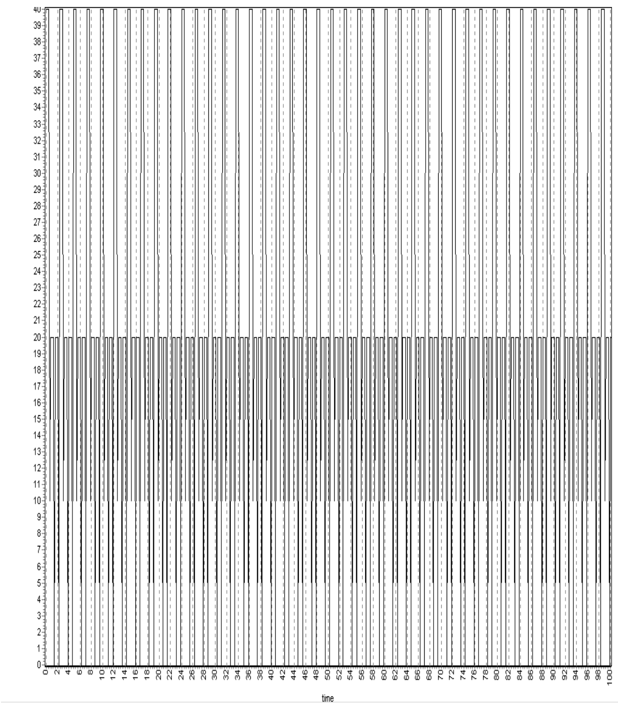


Fig.38.Sum of 2 rectangular periodics

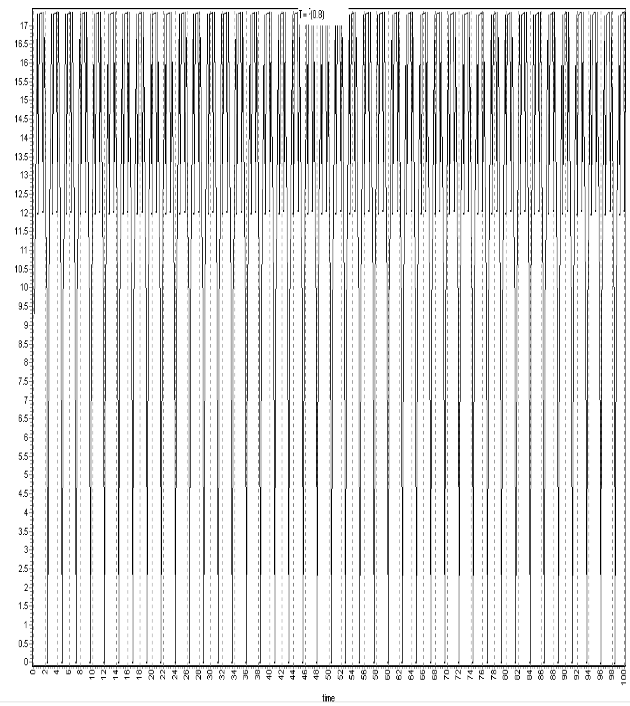


Fig.39. *D*-transf. revealing 1st component

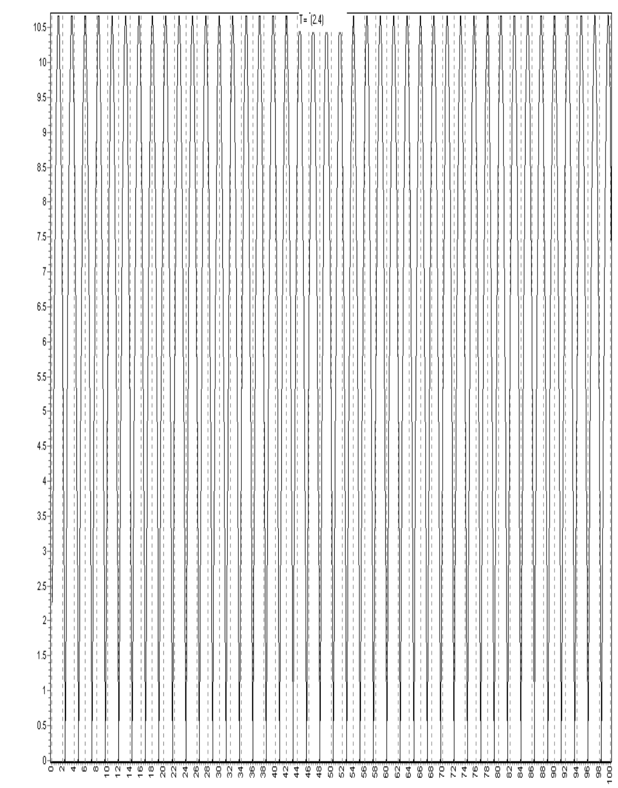


Fig.40.*D*-transf. revealing 2nd component

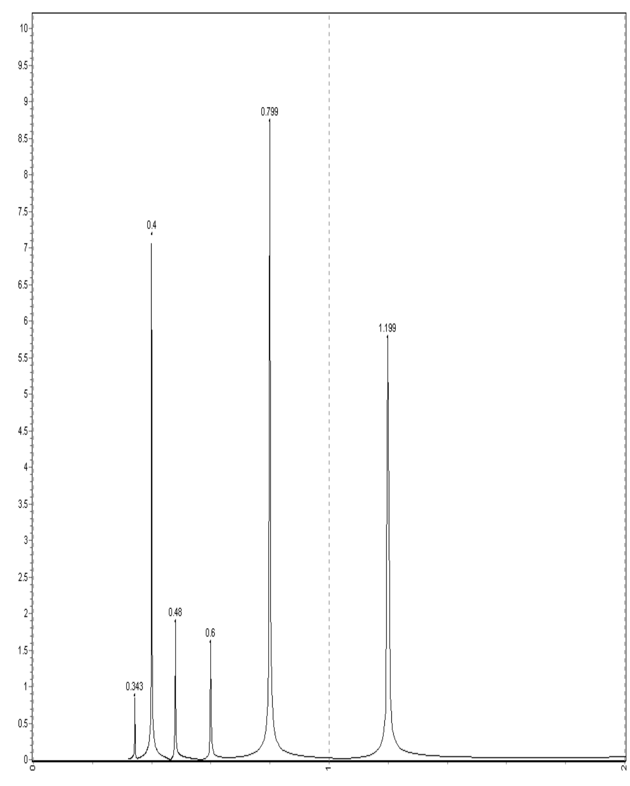


Fig.41.Fourier transform. absolute failure

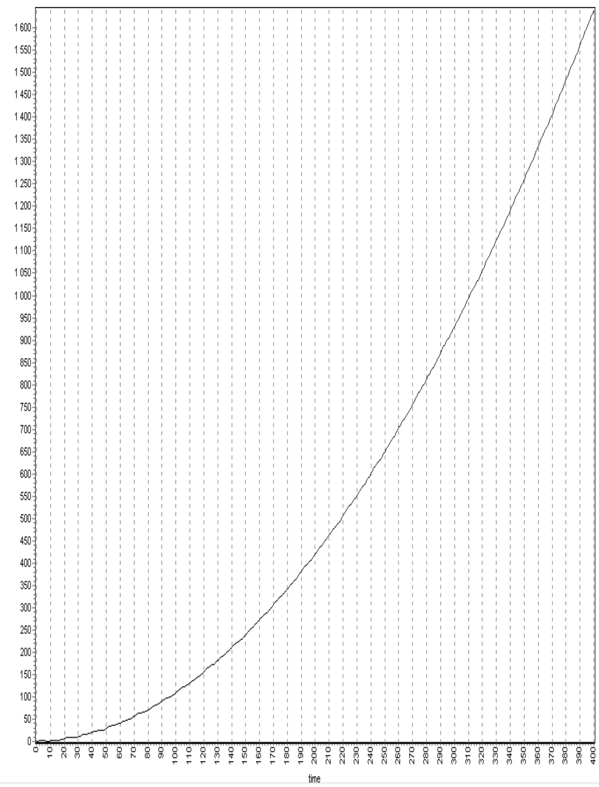


Fig.42. 2 harmonics and parabola

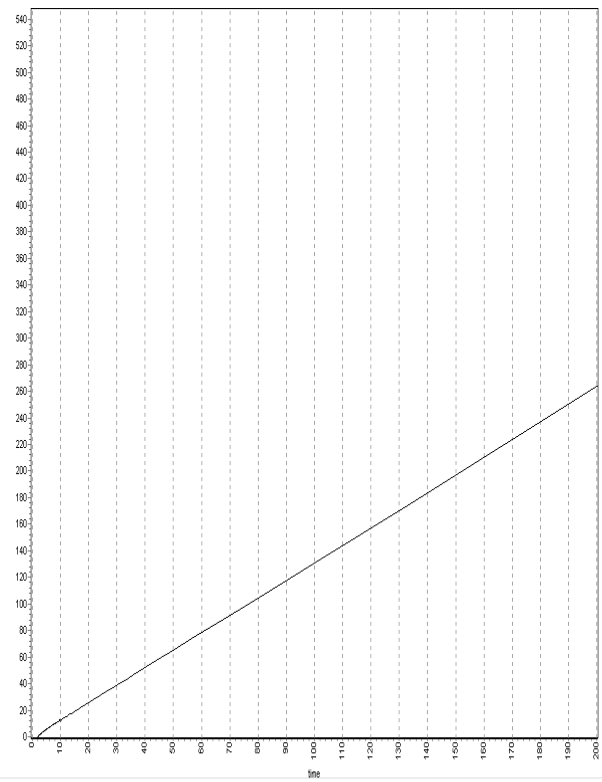


Fig.43. Fourier analysis result

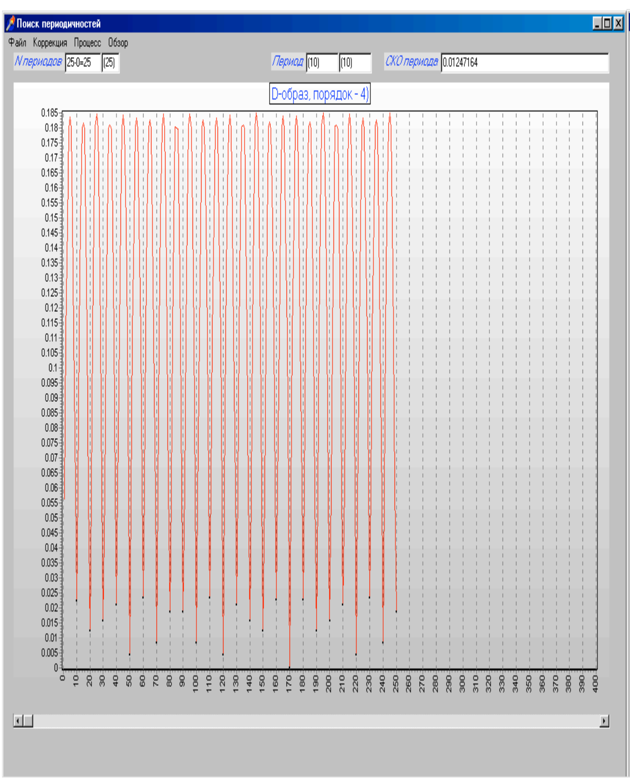


Fig.44. *D*-method revealing 1st harmonic

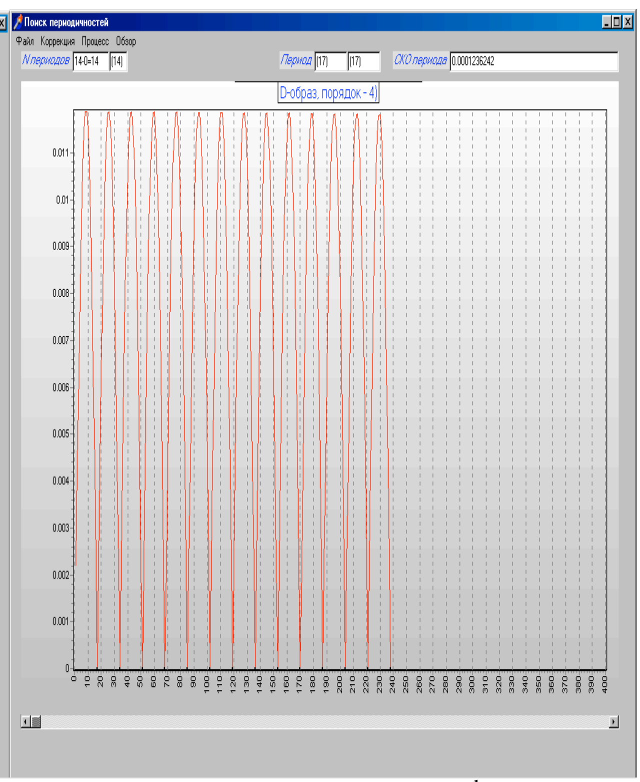


Fig.45. *D*-method revealing 2nd harmonic

R E F E R E N C E S

1. Серебренников М. Г., Первозванский А. А., Выявление скрытых периодичностей, 1965, «Наука», Москва.
2. Вениаминов С. С., Об одном методе вскрытия периодичностей, «Цифровая вычислительная техника и программирование», вып. 5, 1969, «Сов. Радио», г. Москва.
3. Veniaminov S. S., The new family of methods for revealing latent periodicities in signals, Proceedings of 2nd European Conference on Space Debris, 1997, Darmstadt, Germany.
4. Veniaminov S., Dicky V., et al., The new approaches to revealing latent periodicities in the radar and optical signatures of space objects, Proceedings of 4th US/Russian workshop on Space Surveillance, 2000, Washington D. C., USA.
5. Veniaminov S., Dicky V., On revealing latent periodicities in noisy signals, ibid.