Planning of celestial bodies optical search in near-earth space

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The subject of the near-earth astronomy is searching, detecting and investigating the natural and artificial celestial bodies which exist in the near-earth and circumsolar orbits and which are of scientific and practical interest. The information on these bodies is obtained substantially using passive facilities operating in the optical wavelength band. As a rule this facility operates in two modes: in the target designation mode (by ephemerides), or in the autonomous search. It is the second mode that obtainsthe primary data on new celestial bodies being absent from catalogs before or having been lost. The operation in the autonomous mode consists in the purposeful investigation (scanning) of the simple connected or complex connected area of space with the dedicated facility during the allotted time. The searching facility operates in the zone reaching 15 000 sq. degrees and more in angular measure under the dynamic and widely varying optical visibility conditions. Therefore, the scanned area within the zone of operation and the scanning procedure should be optimized. This optimization can be performed based on the following data:

- the search purpose,
- the date of the search session and the location of the dedicated facility,
- the optical visibility conditions in the operation zone during the allotted time,
- the composition and the technical characteristics of the facility.

In the classic setting of the problem the goal of the search is the detection of unknown objects in the investigated area, or ascertainment of the fact that such objects are absent in the said area during the specified period of time. When the search is conducted in the near-Earth space, the search is formulated somewhat differently. As a rule the celestial bodies in the specific orbits are searched and they come into the effective area only by virtue of this fact.

The optimization of the searching procedure is connected with the availability of a priori data about the sought for celestial bodies, as an investigator is interested in not just new celestial bodies, but the celestial bodies of a certain class, and with certain orbital, geometric, reflective and other characteristics. The range of the celestial bodies of interest can be extremely wide, beginning from small-sized elements of space debris made as a result of the human activities in space and ending with the natural bodies of special significance, because of their dangerous approach to the Earth.

Based on these a priori data a certain "conditional catalog of the sought for celestial bodies" can be formed with statistic characteristics corresponding to the supposed orbital parameters and the non-coordinate (non-metric) attributes of the celestial bodies being sought.

Thus, by means of the conditional catalog generation the goal of the autonomous search is specified, which is defined using the information collection

$$K=K(O_1 \ldots O_N, E_1 \ldots E_N, d_1 \ldots d_N),$$

where : N – the number of celestial bodies in the conditional catalog,
O_i – the orbital data of the i^{th} celestial body,
E_i – the non-metric data of the i^{th} celestial body,
d_i – the significance factor of the i^{th} celestial body.

As applied to the i^{th} artificial celestial body in the near-earth orbit $O_{1,}$ these data may contain the following elements:

$$O_i=Q_i(\Omega_i°, i_i°, w_i°, a_i°, e_i°, \tau_i°, MJD_i°, t_i°),$$

where: $\Omega_i°$ - lontitude of the ascending node,
$i_i°$ - orbit inclination,
$w_i°$ - perigee argument,
$a_i°$ - semi-major axis,
$e_i°$ – eccentricity,
$\tau_i°$ - moment of passage through the pericenter,
$MJD_i°$ - MJD day of the elements epoch,
$t_i°$ - UT time of the elements epoch.

The sign "°" indicates that the element value refers to the initial epoch. For the same i^{th} artificial celestial body, the data E_i may have the form of

$$E_i=E_i(Sd_i , Sm_i , ad_i , am_i, Ds_i),$$

where: Sd_i – area of the diffuse reflective surface,
Sm_i – area of the mirror reflective surface,
ad_i – diffuse reflectance,
am_i – mirror reflectance,
Ds_i – data on the artificial celestial body stabilization.

The data of the search session, its duration and the facility location are described by the information set

$$D=D(Date, Ts, B, L, h)$$

where: Date – date of the search session,
Ts - duration of the search session,
B – geodesic latitude of the facility site,
L - geodesic longitude of the site,
h – height of the site over the reference ellipsoid.

Based on the data of sets K and D, the ephemerides of the i^{th} celestial body and its visible angular velocity ω_i within the effective area are determined. Based on the same data the visible angular velocity of the fastest (and slowest) sought celestial body in the conditional catalog, ω_{max} and ω_{min}, is determined.

The optical visibility conditions in the effective area during the allotted time are described using the information set

$$V = V(Sn, Mn, St, At, Sk, Sw)$$

where: Sn – data for calculating the Sun coordinates,
Mn – data for calculating the Moon coordinates and phases,

St – data for calculating the stars background characteristics,
At – data of the atmosphere transparency and stability in the site,
Sk – data for calculating the atmosphere light pollution characteristics,
Sw – data of the snow cover presence in the site.

As a rule the information of the inhabited localities in the area of the facility site (population, remoteness etc.) is used as Sk data.

Based on the data of K, D and V sets, one of the most important characteristics of the i^{th} celestial body is determined – its brightness m_i. Besides, the data of D and V sets are used to build the model of the night sky over the facility site for the allotted time period. Based on these data the distributed background glow brightness Skb and the density of the star background Stb are calculated.

In cases when the specified hardware is of the same type its technical characteristics may be described by the information sets U1, U2 and U3.

The set U1 contains the hardware characteristics that provide acquisition of the celestial body signal:

$$U1=U1(\text{Ö}, f, d, \tau, \varepsilon, \rho, \beta, t_{ac}),$$

where: Ö – lens aperture,
f – lens focal length,
d – lens blur circle diameter,
τ – lens integral transmittance,
ε – detector integral sensitivity,
ρ – detector intrinsic noise,
β – detector pixel size,
t_{ac} – duration of signal accumulation in the detector.

Based on the data of the sets K, D, V, U1 for the i^{th} celestial body, the SNR at the detector output is determined:

$$\psi_i=\psi_i(m_i, \omega_i, Skb, At, \text{Ö}, f, d, \tau, \varepsilon, \rho, \beta, t_{ac}).$$

The set U2 contains the hardware characteristics that provide the signal analysis and decision making:

$$U2=U2(Aa, \delta_t, \delta_s, \delta_e, t_{an}),$$

where: Aa – the data characterizing the signal analysis and decision-making algorithms,
δ_t, δ_s, δ_e – temporal, spatial and energy thresholds set in the analysis equipment,
t_{an} – time spent for the signal analysis and decision-making.

Based on the data of the set U2, the star background density Stb, and the SNR ψ_i for the i^{th} celestial body, the conditional probability of the proper detection is determined:

$$P1i=P1i(\psi_i, Stb, Aa, \delta_t, \delta_s, \delta_e, t_{an}).$$

Besides, based on the data of the sets K, D, V, U1, U2 the number of false alarms is determined:

$$Nfe=Nfe(Skb, Stb, At, Ö, f, d, \tau, \varepsilon, \rho, \beta, t_{ac}, Aa, \delta_t, \delta_s, \delta_e, t_{an}).$$

The set U3 contains the characteristics that provide search capabilities of the facility:

$$U3=U3(Sd, As, S_f, Mt, v_s, a_s),$$

where: Sd – effective area of the facility,

As – data characterizing the scanning algorithm of the investigated area,

S_f – data characterizing the geometric dimensions and the configuration of the detector sensitive surface,

Mt – data characterizing the type and the number of the mounting axes,

v_s – velocity of re-aiming the facility at scanning,

a_s – acceleration of re-aiming the facility at scanning ,

Based on the data of the sets K, D, U1, U3, the summed search capabilities of the facility is determined

$$Q=Q(N_u, As, f, t_{an}, Sf, v_s, \omega_{max}, Ss),$$

where: N_u – number of devices used in search,

Ss – data characterizing the geometric dimensions and the configuration of the scanned area.

When the search capabilities are evaluated, as a rule the measures like sq.degree/h or sr/h are used. At the search session duration Ts the search resource of the facility is

$$QT=Q\times Ts.$$

Based on the data of the sets K, D and the search capabilities Q, the probability of the i^{th} celestial body falling into the hardware field of view and staying in this field for the time of at least t_{an} is determined:

$$P2i=P2i(Q, \omega_i).$$

Then the full probability of the i^{th} celestial body detection is

$$Pi=P1i\times P2i.$$

The calculated full probability of the i^{th} celestial body detection is sufficient for the statement of the problem of the autonomous search optimization. The mathematical expectation of the summed number of new celestial bodies Msb from the conditional catalog K, detected during the time allotted for search, is assumed as a search efficiency criterion. It is obvious that the autonomous search is optimal if Msb is maximum, provided that $Q_T \le Q_T{}^*$ and Nfe≤ Nfe*, i.e.

$$MAX\ Msb \left| \begin{array}{l} K=K^*, \\ Q_T \le Q_T{}^* \\ Nfe \le Nfe^*, \end{array} \right.$$

where the sign "*" marks the values predicted or specified for the next search session.

The autonomous search optimization procedure as such is based on the spatial-temporal discretization of the effective area and the search session. The effective area of the facility Sd is divided into S elementary sections (spatial increments) each of which is a spatial angle with the area of Δs in the angular measure. The search session Ts is divided into T elementary temporal intervals (temporal increments) with the duration of Δt, i.e.

$$S=Sd/\Delta s,$$

$$T=Ts/\Delta t.$$

Then Msb can be expressed as

$$Msb=\sum_{i=1}^{N}\sum_{j=1}^{S}\sum_{\nu=1}^{T}[1-(1-P_{ij\nu})^{qij}]\prod_{j=1}^{S}\prod_{\gamma=1}^{\nu-1}(1-P_{ij\gamma})^{qj\gamma}\prod_{\xi=1}^{j-1}(1-P_{i\xi\nu})^{q\xi\nu} \quad (1)$$

where: $P_{ij\nu}$ – the full probability of the i^{th} celestial body detection in the j^{th} elementary section during the ν^{th} elementary interval,

$q_{j\nu}$ – coefficient characterizing the scanning of the j^{th} elementary section during the ν^{th} elementary interval.

If the section is not scanned, then $q_{j\nu} = 0$, if the scanning of the section is specified, then $q_{j\nu} = 1$.

As was mentioned above the autonomous search optimization comes to a choice of elementary sections subject to scanning and to a sequence of their investigation, whereby the value of Msb is maximum. This means that all values of $q_{j\nu}$ should be defined that will maximize Msb, and thereby all specified search resource Q_T will be used up.

It is evident that the problem of optimization is solved rigorously, if the method of direct search of all versions is used. However, if the conditional catalog is large enough and the effective area is divided into several hundreds of elementary sections, the problem arises that resembles the search of all possible combinations of moves in a chess game. The use of the classic and non-classic methods of optimization, in particular the principle of maximum by L.Pontriagin or the method of dynamic programming by R.Bellman, also appeared to be problematical. Though in some cases the domination principle and acceleration procedures by S. Veniaminov can be useful [7, 8]. Therefore the successive procedure has been developed, which although it does not provide the rigorous solution, it still leads to the results rather close to optimal. This procedure includes the following stages:

1. The stated problem of the autonomous search is analyzed and on its basis the conditional "catalog of sought for celestial bodies" is formed. For many specific search problems the current catalog of space objects available to an observer at the moment of planning the forthcoming session of autonomous search can be taken as the basis of "the conditional catalog of sought for celestial bodies". It is connected with the fact that the naturally arising space environment is characterized by great stability, and the statistic characteristics of a celestial body stream passing through the facility effective area can be regarded as practically unchanged for long periods of time. It is supposed that a new celestial body most likely will be found in anorbit close to those in which there are bodies of a similar class.

If the search task has any peculiarities, the corresponding alterations and corrections can be put in the conditional catalog. The most convenient form of such corrections is the introduction of priorities, in particular in the form of weight coefficients. For example, for the most significant celestial bodies, the coefficient "two" can be introduced. That assumes that not one but two conditional bodies of the same type are in the same conditional orbit. If the search for radically new celestial bodies, not found in the near-earth space earlier, is planned, then the conditional catalog can be arranged on the basis of the supposed physical nature of these bodies and their orbit elements.

Nevertheless, the situation can arise when a priori data are insufficient to form the conditional catalog. This problem is one of the most difficult in the search theory as it supposes the possibility of the presence of the sought for celestial body in any orbits with equal probability. In this case the conditional catalog is formed with the equiprobable distribution of the sought for celestial bodies over the predicted orbit elements and non-metric characteristics.

2. The spatial discretization of the effective area is performed. In doing this the calculation resource allotted for solving the planning task is taken into account. The accuracy of the planning increases as the spatial increment is reduced, however, the demand for the calculation resource rises sharply. It is evident that the size and the configuration of the least possible spatial increment should correspond to the facility field of view. In this case the need to form the scanning algorithm of the spatial increment vanishes, as the increment scanning is brought to one transfer of the field of view, that is defined straight in the process of planning. As the spatial increment increases its size, the configuration should agree with the scanning algorithm, as this algorithm already contains at least several steps. When celestial bodies in high orbits are searched the spatial increment is chosen as a rule in the form of the solid angle of a quadrangular shape constituting a figure, which is delineated by two arc of large circles and two arc of lesser circles. The dimensions of the spatial increment sides are chosen, if possible, multiple to the dimensions of the facility's angular field of view.

3. The temporal discretization of the search session is performed. The value of the temporal increment is defined by

$$\Delta t = \Delta s / Q.$$

4. The celestial bodies from the conditional catalog are "run" through the effective area. The optical visibility conditions are defined in each elementary section, and in each elementary time interval into which the i^{th} celestial body falls. The facility operation mode is set which has been planned for these conditions beforehand, and which provides the fulfillment of the requirement $Nfe \leq Nfe^*$. $P_{ij\nu}$ is defined for each celestial body.5. Following the "run" of all bodies from the conditional catalog, the mathematical expectation of the number of detected celestial bodies $Mj\nu$ is determined for each section and each time interval, provided that the section and the time interval has been specified for scanning. Here

$$M_{j\nu}=\sum_{i=1}^{N} P_{ij\nu}$$

The results are given in tabular form.

№ sb	Elementary time intervals													
	ν=1				ν=2				…		ν=T			
	Elementary sections				Elementary sections						Elementary sections			
	j=1	j=2	…	j=S	j=1	j=2	…	j=S	…	…	j=1	J=2		J=S
i=1	P_{111}	P_{121}		P_{1S1}	P_{112}	P_{122}		P_{1S2}			P_{11T}	P_{12T}		P_{1ST}
i=2	P_{211}	P_{221}		P_{2S1}	P_{212}	P_{222}		P_{2S2}			P_{21T}	P_{22T}		P_{2ST}
…														
i=N	P_{N11}	P_{N21}		P_{NS1}	P_{N12}	P_{N22}		P_{NS2}			P_{N1T}	P_{N2T}		P_{NST}
Mjν	M_{11}	M_{21}		M_{S1}	M_{12}	M_{22}		M_{S2}			M_{1T}	M_{2T}		M_{ST}

6. The elementary section and the elementary time interval with the minimum Mjν is defined. This section in this time interval is specified for scanning.

7. All Pijν are re-calculated taking into account the expected scanning results of the section chosen in the previous step.

8. All Mjν are re-calculated, a new table is made.

9. According to the re-calculated table, a new section and a new interval with the maximum Mjν are defined. This new elementary section in the new elementary time interval is also specified for scanning. Note that under certain conditions the section that already has been scanned in some previous time interval, can be specified for scanning.

Pijν and Mjν are re-calculated until all search resources will be spent. During the re-calculation, the most promising sections are selected and the most expedient procedure of their scanning is set. In fact, the operation of the facility specified for the autonomous search during the allotted time is optimally "tuned" for the given space environment and the predicted optical visibility conditions.

Thus, the above discussion has shown the principal possibility of full formalization and, hence, automation of the planning process of new celestial bodies optical search in the near-Earth space. Unfortunately, the formalization of the particular tasks in the particular conditions and using the particular facility is associated with certain difficulties, especially at the stages of obtaining the reliable values of ψ_i, P1, P2, $P_{ij\nu}$, Nfe and Q. However, taking into consideration the fact that in many cases an observer has experimental data on the particular facility, and investigation and testing in the laboratory and in field, these difficulties seem quite surmountable. If the experimental data are insufficient, it is expedient to use the possibilities of computer modeling in the separate stages of the planning process.

With the increase of the calculation resources allotted for planning, the spatial and temporal specifications of the search process may increase, and, thereby, the accuracy of planning rises. Besides, the increased calculation resource takes into consideration a larger number of affecting factors. This refers, in particular, to astroclimatic conditions, the technical characteristics of the hardware, including that of different types, the analysis and decision-making algorithms, scanning algorithms, and others.

In conclusion, it may be said that although the procedure has been developed for planning the search for new, earlier unknown celestial bodies, it may appear to be useful for planning observations of the bodies with sufficient volume of metric and non-metric data.

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