SPACE DEBRIS AS AN EPIDEMIC

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Introduction:

Since seminal papers by Kessler and others appeared[2], space stakeholders have accepted the existence of a tipping point in space debris density beyond which debris will proliferate exponentially. There is active and continuous debate over whether the critical density has been achieved. Using principles of epidemiology, this paper explains why there is no tipping point. The density of fragments will increase rapidly from a current initial state under all circumstances, leading to a new, more hazardous equilibrium.

This is not a unique or new observation. This behavior has been well recognized, if not well publicized or accepted for several years. Dr. David Talent, Kessler's former colleague noted this behavior in analyses conducted with his PODEM model[3]. ESA models such as MASTER and DREAM do not demonstrate precipitous growth. Kitazawa has compared several nonlinear models, and none predict a tipping point. [4]

Background

In 1798, Thomas Robert Malthus outlined an exponential model of population growth. Analogously, linearized theories of debris growth exhibit a critical value of debris density. Exponential catastrophes are characteristic of approximations that simplify the solution of physical problems by allowing superposition of many independent solutions of the simplified equations. Being able to add solutions allows one to solve problems with different sets of constraints, called boundary and initial conditions. The simplification fails when the amplitudes of the solutions grow large. Large amplitudes are inhibited by phenomena that dissipate energy. Malthus approximations are not valid when effects are large.

Malthus contemporary, Pierre Verhulst realized that there are forces at work to prevent exponential growth and correctly postulated that these forces increase in direct proportion

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[2] Kessler, D.J. and Cour-Palais, B.G., "Collision Frequency of Artificial Satellites: The Creation of a Debris Belt," Journal of Geophysical Research, Vol. 83, No. A6, 1978, pp 2637-2646

[3] Talent, D.L., "Analytic Model for Orbital Debris Environment Management," J. Spacecraft & Rockets, Vol 29, 1992, pp 508-513.

[4] Fukushige, S., Akaoshi, Y., Kitazawa, Y., and Goka, T., "Comparison of Debris Environment Models: ORDEM2000, MASTER2001, AND MASTER2005, Ishikawajima-Haria Heavy Industries Co., Ltd, Engineering News, Vol 40, No. 1, Feb 2007

to the ratio of excess population to total population. [5] In simple terms, for space debris this means that the population growth depends not just on the size of the population but also on the rate of growth. The faster the growth and the greater the population, the more slowly the population will grow until it stabilizes at a new state appropriate to the new conditions.

This reasoning applies to human populations, to the growth of bacteria, and to the balance between prey and predators. In the book A Beautiful Math, the author presents this in the frame work of game theory. Populations that are all passive (prey-like) or all predatory are unstable. They will die out either through starvation or cannibalistic predation. There is a stable state, usually with many more prey than predators.

Infinities plague virtually all of physics and mathematics. Scholars recognize that infinities are specious and seek to uncover the reasons for unrealistic catastrophes, extending theories until the next road block. Paul Dirac is probably the most famous, often cited for telling Princeton Physicist Ed Witten that the most important challenge in physics was "to get rid of infinity." Which he helped overcome through renormalization in quantum electrodynamics. Dirac is buried near Newton in Westminster Abbey, which is testimony to his insight.[6]

These lead to the conclusion that the classical Kessler model and the catastrophes it predicts are Malthusian, omitting the rate dependencies and the evolution to a Verhulst equilibrium.

Analysis Evolution

The original equations attributed to Kessler and colleagues are:

$$dB/dt = Si\ \sigma i\ V\ \Delta U + Si\ Sf\ \sigma f\ V\ \Delta U \quad (1)$$
[7]

This includes breakups due to collisions of intact spacecraft with debris, collisions between debris particles, and the fact that the number of intact satellites decreases as the number of fragments increases.

Faranella and Cordelli conceived a model that estimates both the rate of change of breakup fragments and that of intact satellites.[8] LaFleur[9] expands Faranella-Cordelli

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[5] Verhulst, P., "Notice sur la loi que la poplation poursuit dans son accroissement," Correspondance methdmatique et physique 10, 113-121 (publicly available at no cost from Google Books).
[6] **The Strangest Man: The Hidden Life of Paul Dirac, Mystic of the Atom [Hardcover]** Graham Farmelo
[7] CRITICAL NUMBER OF SPACECRAFT IN LOW EARTH ORBIT: USING SATELLITE FRAGMENTATION DATA TO EVALUATE THE STABILITY OF THE ORBITAL DEBRIS ENVIRONMENT Donald J. Kessler(1), Phillip D. Anz-Meador(2), 1978
[8] Farinella, P. and Cordelli, A., "The Proliferation of Orbiting Fragments: A Simple Mathematical Model," Science & Global Security, Vol. 2, No. 4, 1991, pp 365-378

formalism. Considering natural orbital decay, the rate at which spacecraft are launched, collisions among debris elements, and collisions between productive satellites:

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\begin{aligned}
\frac{dN}{dt} &= \left[a + b\sin\left(ct + d\right)\right] - \frac{N}{f + g\sin(ht + k)} - xnN - 2yN^2 \\
\frac{dn}{dt} &= \beta\left[a + b\sin\left(ct + d\right)\right] - \frac{n}{p + q\sin(ht + k)} + \alpha xnN + \gamma yN^2 - 2zN^2
\end{aligned}
\tag{2}
$$

One need not delve deeply into the calculus in order to understand what these equations mean. N is the number of active satellites and "n" is the number of debris fragments. The first term in each equation is the rate at which satellites are launched or debris deployed rather than generated in collisions. The second terms are decay rates due to atmospheric drag. The periodic variations estimate variable launch rates and solar cycle influences on energy dissipation. The third terms correspond to collisions between intact satellites and debris, which decreases the number of satellites and increases the number of fragments. Several fragments may be created as the result of one collision. The last terms are fratricidal collisions between like objects. LaFleur assumes that very small fragments are not a threat; therefore, threatening fragments that collide with each other annihilate each other by producing fragments too small to be threats.

This system of equations permits stable evolutions. The original Kessler equations do not unless the non-collisional production rates are negative, which corresponds to debris removal. Hence, cascading is inevitable, and only the rate of exponentiation is controllable. The equilibrium state defined by zero rate of growth of debris is the crest of a hill in parameter phase space. Perturbations drive the system away from equilibrium. LaFleur estimates the coefficients in these equations and conducts a rigorous stability assessment, linearizing about the equilibrium state. He finds that the equilibrium state is unconditionally stable, the depth of a valley in parameter phase space.

Unfortunately, the equilibrium state that LaFleur estimates is millions of fragments and less than 2000 active satellites. In other words, the launch rate would have to be much less than now in order to stay in balance with decays.

Discussion:

LaFleur simplifies the problem by omitting the temporal variation of launch and decay rates to a normalized form:

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[9] LaFleur, J.M., Extension of a Simple Model for Orbital Debris Proliferation and Mitigation, AAS 11-173, Spaceflight Mechanics 2011, Proceedings of the 21st AAS/AIAA Flight Mechanics Meeting, Feb 13-17, 2011,. New Orleans, LA.

$$\frac{dM}{ds} = 1 - \chi\rho M m$$
$$\frac{dm}{dt} = 1 - m + \chi M m \qquad (3)$$

Where M is the normalized number of satellites, m is the normalized number of debris fragments, X is the product of the yearly satellite launch rate, the square of the average decay time of debris fragments, the number of fragments generated when a satellite experiences a collision, and the collision rate per satellite per year, ρ is the ratio of fragments deployed upon launch to fragments generated when a satellite experiences a collision. This equation possesses an equilibrium solution:

$$M_e = \frac{1}{\chi(1+\rho)}$$
$$m_e = \frac{(1+\rho)}{\rho} \qquad (4)$$

$$Me = \frac{1}{\chi\cdot(\rho + 1)} \qquad me = \frac{(1 + \rho)}{\rho}$$

The real parts of the eigenvalues of the system linearized about equilibrium are always negative for feasible values of X; hence the system of equations is unconditionally stable. This is in contrast to the Kessler equations, which are unconditionally unstable.

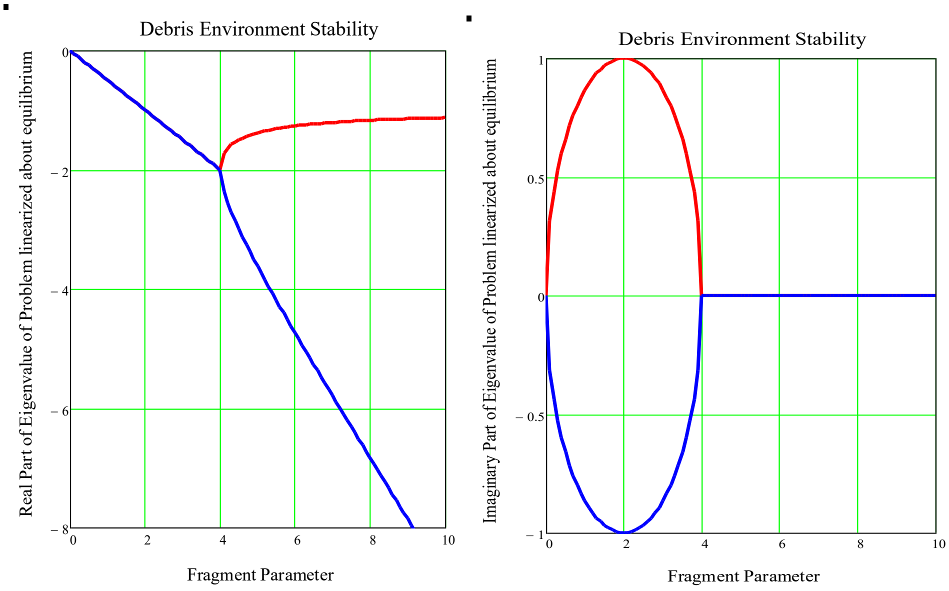

Figure 1: Stability characteristics. Below χ=4, there are two possible solutions oscillatory with different phases and decaying amplitude envelopes. Above that there are

two continuously decaying solutions, one decaying very rapidly and the other more slowly. All solutions are unconditionally stable.

The critical processes that render the environment stable are:

- Establishing a threshold of minimum fragment sizes below which the fragments cannot inflict disabling damage.
- Considering that any debris in low Earth orbit will eventually reenter due to atmospheric resistance.
- Recognizing that collisions between debris fragments and between satellites create much different amounts of debris.

Conclusion: The consequences of this description of the evolution of the debris environment are that the equilibrium state might allow many fewer satellites than are required to fulfill the potential of near Earth space.   However, the properly normalized mathematical statement of the problem confirms intuition that the equilibrium satellite population is highest when:

- Fragments decay from orbit quickly
- Satellites are constructed to generate the least number of fragments possible
- Collision rates are diminished by minimizing satellite cross sections, and the launch rate is controlled to minimize capricious missions or those with little quantifiable benefit.

Just as pendulums do not swing forever because of friction in the pivots, structures excited at resonance to not experience gyrations with infinite amplitude, and plagues do not kill everyone, space debris will not proliferate exponentially to infinity.